

## **5. CONCRETE STRUCTURES**

Reinforced and prestressed concrete are used extensively in bridge projects. In addition to general design guidance and information on detailing practices, this section contains three design examples: a three-span reinforced concrete slab superstructure, a 63 inch pretensioned I-beam, and a three-span post-tensioned concrete slab superstructure.

### **5.1 Materials**

For most projects, conventional materials should be specified. Standard materials are described in two locations: *MnDOT Standard Specifications for Construction* (MnDOT Spec.) and *Bridge Special Provisions*.

If multiple types of concrete or reinforcement are to be used in a project, it is the designer's responsibility to clearly show on the plans the amount of each material to be provided and where it is to be placed.

#### **5.1.1 Concrete**

MnDOT Specs. 2461 (cast-in-place) and 2462 (precast) identify and describe concrete mix types used by MnDOT. Based on their properties, different mixes are used for the various structural concrete components. Table 5.1.1.1 identifies the standard MnDOT concrete mix types to be used for different bridge components.

Four characters are typically used to identify a concrete mix. The first character designates the type of concrete (based on air entrainment requirements). The second character identifies the grade of concrete based on multiple characteristics (intended use, max. w/c ratio, max. cementitious content, max. supplementary cementitious materials, min. concrete strength, etc). The third character is the upper limit for the slump in inches. The fourth character identifies the coarse aggregate gradation. There are some exceptions to the above: MnDOT designed mixes (such as 3U17A low slump concrete), job mixes (JM) for box girders, and high performance concrete (HPC) mixes for bridge decks and slabs.

For HPC mixes, the first and second characters follow the description above. For monolithically poured decks, these are followed by either "HPC-M" or "LCHPC-M" (where the LC designates low cement). For decks that will receive a separate wearing course, these are followed by either "HPC-S" or "LCHPC-S" (where the LC designates low cement). For job mixes, the first character designates the type of concrete as above, but is followed by "JM" for mixes that will be determined by the Contractor.

In general, the standard concrete design strength is 4 ksi, and air entrained concretes are to be used for components located above footings and pile caps to enhance durability.

**Table 5.1.1.1 Design Concrete Mix Summary**

Location/Element	MnDOT Concrete Mix Designation	Design Compressive Strength (ksi)	Maximum Aggregate Size (in)
Cofferdam seals	1X62	5.0	1
Cast-in-place concrete piles and spread footing leveling pads	1P62	3.0	2
Drilled shafts and rock sockets	1X62 3X62	5.0 5.0	1 1
Footings and pile caps, except not for partially exposed abutment footings behind MSE walls.	1G52	4.0	1 ½ *
Abutment stems, wingwalls, pier columns, pier struts, pier caps, and moment slabs. Also includes partially exposed abutment footings behind MSE walls.	3B52	4.0	1 ½ *
Integral abutment diaphragms and pier continuity diaphragms	Same mix as used in deck	4.0	1
Pretensioned superstructures	1W82 or 3W82	5.0 – 9.5 at final 4.5 – 8.0 at initial	1
Cast-in-place and precast box girders	3JM	6.0 or higher	1
Monolithic decks and slabs	3YHPC-M, 3YLCHPC-M or 3Y47-M	4.0	1
Decks and slabs that will receive a 2 inch concrete wearing course	3YHPC-S, 3YLCHPC-S or 3Y47-S	4.0	1
Barriers, parapets, medians, sidewalks, and approach panels	3S52	4.0	1
Concrete wearing course	3U17A	4.0	5/8
MSE wall panels, PMBW blocks, and noise wall panels	3Y82	4.0	1
Cast-in-place wall stems	3G52	4.5	1 ½ *
Precast box culverts, arches, and 3-sided structures	3W82	5.0 or higher	1*

\* For determination of  $s_{xe}$  per LRFD 5.7.3.4.2, use max aggregate size  $a_g = 3/4"$

**Reinforced Concrete Sections**

Base concrete modulus of elasticity computations on a unit weight of 0.145 kcf. Use a unit weight of 0.150 kcf for dead load calculations.

For structural modeling (determining design forces and deflections), use gross section properties or effective section properties. For redundant structures with redundant and nonprismatic members, model with nonprismatic elements.

**[5.4.2.4]**

For reinforced concrete elements, use:  $E_c = 120,000 \cdot K_1 \cdot w_c^2 \cdot f'_c{}^{0.33}$

For checks based on strength (design of reinforcement, maximum reinforcement), use conventional strength methods (reinforcement yielding, Whitney equivalent stress block, etc.).

For checks based on service loads (fatigue, crack control, etc.), use cracked sections with reinforcing steel transformed to an equivalent amount of concrete.

**Mass Concrete Elements**

Identify mass concrete elements by determining if their geometry meets the criteria listed in Table 5.1.1.2. If the mass concrete designation is required, include any associated standard notes and mass concrete pay item. In addition, identify all required elements as mass concrete anywhere their concrete mix designation is depicted in the Final Bridge Plans.

**Table 5.1.1.2 Mass Concrete Dimension Requirements**

Concrete Element	Least Dimension*
Buried Footings and Pier Crash Struts	≥ 60 in.
All Other Concrete Elements	> 48 in.

\* When the concrete element has variable dimensions on a given axis, use the largest variable dimension for the least dimension determination.

**Prestressed Concrete Elements**

When computing section properties, use a modular ratio of 1 for the prestressing strands.

For pretensioned beams (M, MH, MN, MW, and RB) fabricated using concrete with a final concrete strength,  $f'_c$ , greater than 6.0 ksi, compute the modulus of elasticity using the ACI 363 equations below for the concrete at all stages of strength:

$$E_{ci} = 1265 \cdot \sqrt{f'_{ci}} + 1000 \quad (\text{where } f'_{ci} \text{ and } E_{ci} \text{ are in ksi})$$

$$E_c = 1265 \cdot \sqrt{f'_c} + 1000 \quad (\text{where } f'_c \text{ and } E_c \text{ are in ksi})$$

For all other pretensioned and post-tensioned elements, compute the modulus of elasticity using AASHTO LRFD Equation 5.4.2.4-1, with  $K_1 = 1$  and  $w_c = 0.150$  kcf.

For both pretensioned and post-tensioned elements, use a unit weight of 0.155 kcf for dead load calculations.

Table 5.1.1.3 summarizes concrete properties for analysis and design:

**Table 5.1.1.3  
Concrete Properties**

Parameter	Equation/Value
Unit Weight	Reinforced Concrete Elements: $w_c = 0.145 \text{ kcf}$ for calculation of $E_c$ $w_c = 0.150 \text{ kcf}$ for dead load calculation Pretensioned and Post-tensioned Elements: $w_c = 0.150 \text{ kcf}$ for calc. of $E_c$ (except pretensioned beams with final concrete strength $f'_c > 6 \text{ ksi}$ ) $w_c = 0.155 \text{ kcf}$ for dead load calculation
Modulus of Elasticity	Pretensioned Beams: Where $f'_c \leq 6 \text{ ksi}$ : $E_{ci} \text{ (ksi)} = 120,000 \cdot K_1 \cdot w_c^2 \cdot f'_{ci}{}^{0.33}$ $E_c \text{ (ksi)} = 120,000 \cdot K_1 \cdot w_c^2 \cdot f'_c{}^{0.33}$ Where $f'_c > 6 \text{ ksi}$ : $E_{ci} \text{ (ksi)} = 1265 \cdot \sqrt{f'_{ci}} + 1000$ $E_c \text{ (ksi)} = 1265 \cdot \sqrt{f'_c} + 1000$ All Other Concrete Elements: $E_c \text{ (ksi)} = 120,000 \cdot K_1 \cdot w_c^2 \cdot f'_c{}^{0.33}$
Thermal Coefficient	$\alpha_c = 6.0 \times 10^{-6} \text{ in/in/}^\circ\text{F}$
Shrinkage Strain	Reinf. Conc.: $\epsilon_{sh} = 0.0002$ @ 28 days and $0.0005$ @ 1 year Prestressed Concrete: per LRFD Art. 5.4.2.3
Poisson's ratio	$\nu = 0.2$

**5.1.2 Reinforcing Steel**

Reinforcing bars shall satisfy MnDOT Spec 3301. ASTM A615 Grade 60 deformed bars (black or epoxy coated) should be used in most circumstances. In some cases, Grade 75 stainless steel bars will be required in the bridge deck and barrier (see Tech. Memo No. 17-02-B-01 *Requirements for the Use of Stainless Steel Reinforcement in Bridge Decks & Barriers*). Use  $f_y = 75 \text{ ksi}$  when designing with stainless steel bars. Always use stainless steel (Grade 75) for the connecting bar between approach panel and end diaphragm at integral and semi-integral abutments.

In specialized situations and with the approval of the State Bridge Design Engineer, welding to reinforcement may be used. ASTM A706 Grade 60 bars must be used for applications involving welding.

The modulus of elasticity for mild steel reinforcing ( $E_s$ ) is 29,000 ksi.

All reinforcement bars, except stainless steel bars and bars that are entirely embedded in footings, shall be epoxy coated. Note that for footings that are not entirely buried, such as for abutments behind mechanically stabilized earth walls, reinforcing bars must be epoxy coated.

### **5.1.3 Reinforcement Bar Couplers**

Contractors select reinforcement bar couplers that meet the requirements stated in MnDOT Spec. 2472.3.D.2.

In general, the couplers need to:

- Provide a capacity that is 125% of the nominal bar capacity.
- Be epoxy coated.
- Satisfy fatigue testing requirements of NCHRP Project 10-35 (12 ksi).

### **5.1.4 Prestressing Steel**

Note: Portions of this article have been superceded. See Memo to Designers #2021-01 for guidance on use of higher strength prestressing strands with  $f_{pu} = 300$  ksi.

Uncoated low-relaxation 7-wire strand or uncoated deformed, high-strength bars are acceptable prestressing steels. Strands shall conform to ASTM A416. Bars shall conform to ASTM A722.

Use the following properties for prestressing steel:

Tensile strength:  $f_{pu} = 270$  ksi for strands

$f_{pu} = 250$  ksi for bars

Yield strength:  $f_{py} = 243$  ksi for strands

$f_{py} = 120$  ksi for bars

Elastic Modulus:  $E_p = 28,500$  ksi for strands

$E_p = 30,000$  ksi for bars

Standard 7-wire prestressing strand area,  $A_{ps}$ :

$\frac{3}{8}$ " diameter strand: 0.085 in<sup>2</sup>/strand

$\frac{1}{2}$ " diameter strand: 0.153 in<sup>2</sup>/strand

0.6" diameter strand: 0.217 in<sup>2</sup>/strand

### **5.1.5 Post-tensioning Hardware**

For post-tensioned concrete bridges, open ducts must be used for tendon passageways through the superstructure. Longitudinal ducts are typically 3 to 4 inches in diameter and must be sufficiently rigid to withstand the loads imposed upon them. The preferred material for longitudinal ducts is corrugated plastic (HDPE). Transverse ducts are typically smaller, containing from 1 to 4 strands. Because the transverse ducts are relatively

close to the top of the deck with heavy applications of corrosive de-icing chemicals, corrugated plastic ducts are required. The anchor head is typically galvanized or epoxy coated based on project needs. Discuss the protection requirements with the State Bridge Design Engineer.

Tendon anchorage devices are required at the ends of each duct. Anchorages should be shown and indicated on the drawings. Detailing is unnecessary because the post-tensioning supplier will provide these details in the shop drawings for the post-tensioning system. Designers must consider the local zone anchorage reinforcement (typically spiral reinforcement) provided by potential suppliers to allow adequate room for the general zone reinforcement designed and detailed in the bridge plans.

## **5.2 Reinforcement Details**

Practices for detailing a variety of reinforced concrete elements are presented in this section. These include standard concrete cover and bar spacing dimensions, plus a variety of specific design and detailing instructions.

Reinforcing details are intended to provide a durable structure with straightforward details. Details must be constructible, allowing steel to be placed without undue effort, and provide adequate clear cover and adequate space between reinforcement to permit the placement of concrete.

### **5.2.1 Minimum Clear Cover and Clear Spacing**

The minimum clear cover dimension to reinforcement varies with the location in the bridge. It varies with how the component is constructed (precast, cast in forms, cast against earth) and the exposure the element has to de-icing salts. In general, minimum covers increase as control over concrete placement decreases and as the anticipated exposure to de-icing salts increases. Following is a list of structural components and the corresponding minimum clear cover. For components that are not listed, a 2" minimum clear cover is required unless it is shown differently in the Bridge Office standards.

#### **Foundations**

##### *Top Bars*

- Minimum clear cover is 3 inches.

##### *Bottom Bars, Spread Footing*

- Minimum clear cover to the bottom concrete surface is 5 inches.
- Minimum clear cover to the side concrete surface is 3 inches.

*Bottom Bars, Pile Cap w/ Pile Embedded 1 foot*

- Rest directly on top of trimmed pile.

*Bottom Bars, Pile Cap Alone or Where Pile Cap is Cast Against a Concrete Seal, w/ Pile Embedded More Than 1 foot*

- Minimum clear cover is 3 inches to bottom of pile cap.

**Abutments, Piers, and Pier Crash Struts**

- Standard minimum clear cover for all bars is 2 inches (vertical and horizontal).
- At rustications, the minimum horizontal clear cover varies with the size of the recess. For recesses less than or equal to 1 inch in depth and less than or equal to 1 inch in width, the minimum clear cover is 1.5 inches. For all other cases, the minimum clear cover is 2 inches.
- Minimum clear distance between reinforcement and anchor rods is 2 inches.
- In large river piers with #11 bars or larger that require rebar couplers, minimum clear cover to bars is 2.5 inches.

**Decks, Slabs, Sidewalks, and Raised Medians***Top Bars, Roadway Bridge Deck or Slab, Sidewalk, and Raised Median*

- Minimum clear cover for epoxy coated bars to the top concrete surface is 3 inches.
- Minimum clear cover for stainless steel bars to the top concrete surface is 2½" inches for monolithic decks, 3" for partial depth decks with a wearing course, and 4" for concrete box girder decks.
- Minimum horizontal clear cover is 2 inches. In the bridge plan, detail bars with an edge clear cover of 2 inches, but compute bar length assuming 2½" clear.

*Top Bars, Pedestrian Bridge Deck*

- Minimum clear cover to the top concrete surface is 2 inches.

*Bottom Bars, Deck*

- Minimum clear cover to the bottom concrete surface is 1 inch.
- Minimum horizontal clear cover from the end of the bar to the face of the concrete element is 4 inches.
- Minimum horizontal clear cover from the side of a bar to the face of the concrete element is 2 inches.

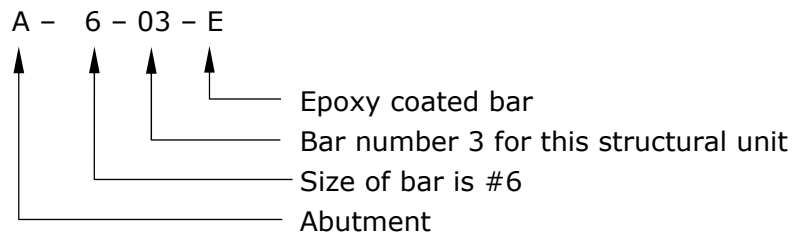
*Bottom Bars, Slab*

- Minimum clear cover to the bottom concrete surface is 1.5 inches.
- Minimum horizontal clear cover from the end of the bar to the face of the concrete element is 4 inches.
- Minimum horizontal clear cover from the side of a bar to the face of the concrete element is 2 inches.

**5.2.2 Reinforcing Bar Lists**

For numbering of reinforcing bars, the first character is a unique alpha character for the given structural element. The first one or two digits of the bar mark indicate the U.S. Customary bar size. The last two digits are the bar's unique sequential number in the bar list for that substructure or superstructure unit. A suffix "E" indicates the bar is epoxy coated, "G" indicates the bar is galvanized, "S" indicates the bar is stainless steel, "M" indicates the bar is epoxy coated 4% chromium, "F" indicates the bar is Glass Fiber Reinforced Polymer (GFRP), "Y" indicates a Grade 75 epoxy coated bar, and "Z" indicates a Grade 75 plain bar.

For example, an A603E bar could be decoded as follows:



The cross-sectional areas, diameters, and weights of standard reinforcing bars are provided in Table 5.2.2.1.

**Table 5.2.2.1  
Reinforcing Steel Sizes and Properties**

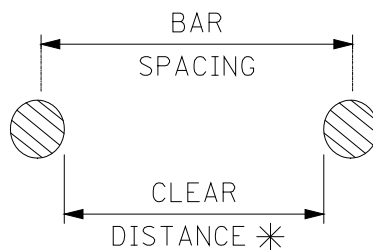
U.S. Customary Bar Size	Area of Bar (in <sup>2</sup> )	Diameter of Bar (in)	Weight of Bar (lb/ft)
#3	0.11	0.375	0.376
#4	0.20	0.500	0.668
#5	0.31	0.625	1.043
#6	0.44	0.750	1.502
#7	0.60	0.875	2.044
#8	0.79	1.000	2.670
#9	1.00	1.128	3.400
#10	1.27	1.270	4.303
#11	1.56	1.410	5.313
#14	2.25	1.693	7.650
#18	4.00	2.257	13.60

Table 5.2.2.2 lists the reinforcing steel area provided (per foot) for different sized bars with different center to center bar spacings.



**Table 5.2.2.2**  
**Average Area per Foot Width Provided by Various Bar Spacings (in<sup>2</sup>/ft)**

Bar Size Number	Nominal Diameter (in)	Spacing of Bars in Inches												
		3	3.5	4	4.5	5	5.5	6	7	8	9	10	11	12
3	0.375	0.44	0.38	0.33	0.29	0.26	0.24	0.22	0.19	0.17	0.15	0.13	0.12	0.11
4	0.500	0.80	0.69	0.60	0.53	0.48	0.44	0.40	0.34	0.30	0.27	0.24	0.22	0.20
5	0.625	1.24	1.06	0.93	0.83	0.74	0.68	0.62	0.53	0.47	0.41	0.37	0.34	0.31
6	0.750	1.76	1.51	1.32	1.17	1.06	0.96	0.88	0.75	0.66	0.59	0.53	0.48	0.44
7	0.875	2.40	2.06	1.80	1.60	1.44	1.31	1.20	1.03	0.90	0.80	0.72	0.65	0.60
8	1.000	3.16	2.71	2.37	2.11	1.90	1.72	1.58	1.35	1.19	1.05	0.95	0.86	0.79
9	1.128	4.00	3.43	3.00	2.67	2.40	2.18	2.00	1.71	1.50	1.33	1.20	1.09	1.00
10	1.270	---	4.35	3.81	3.39	3.05	2.77	2.54	2.18	1.91	1.69	1.52	1.39	1.27
11	1.410	---	---	4.68	4.16	3.74	3.40	3.12	2.67	2.34	2.08	1.87	1.70	1.56



\* Per LRFD 5.10.3.1.1, the minimum clear distance between bars in a layer shall be the greatest of:

- 1) 1.5 times the nominal diameter of the bar
- 2) 1.5 times the maximum size of the coarse aggregate \*\*
- 3) 1.5 inches

\*\* Per the current edition of *MnDOT Standard Specifications for Construction*

The weight of spiral reinforcement on a per foot basis is provided in Table 5.2.2.3. The standard spiral reinforcement is  $\frac{1}{2}$  inch diameter with a 3 inch pitch. When selecting the size of round columns, use outside dimensions that are consistent with cover requirements and standard spiral outside diameters.

Figure 5.2.2.1 through 5.2.2.6 contain development length (Class A lap) and tension lap splice design tables for epoxy coated, plain uncoated, and stainless steel reinforcement bars. Knowing the bar size, location, concrete cover, bar spacing, and class of splice, designers can readily find the appropriate lap length. The tables are based on 4 ksi concrete.

Figure 5.2.2.7 contains development length tables for bars with standard hooks. Values are provided for epoxy coated, plain uncoated, and stainless steel reinforcement bars. Standard hook dimensions are also included.

Figure 5.2.2.8 contains graphics that illustrate preferred and acceptable methods for anchoring or lapping stirrup reinforcement. All stirrups must be anchored by hooking around longitudinal reinforcement. Detail closed double stirrups with a Class B lap. Also included in Figure 5.2.2.8 are stirrup and tie hook dimensions and a table showing minimum horizontal bar spacings for various concrete mixes.

**Table 5.2.2.3**  
**Weight of Spiral Reinforcement**

O.D. SPIRAL (in)	WEIGHTS IN POUNDS PER FOOT OF HEIGHT			
	<sup>3</sup> / <sub>8</sub> " DIA. ROD		<sup>1</sup> / <sub>2</sub> " DIA. ROD	
	6" PITCH (lb/ft)	F (lb)	3" PITCH (lb/ft)	F (lb)
24	4.72	7.1	16.79	12.60
26	5.12	7.7	18.19	13.65
28	5.51	8.3	19.59	14.70
30	5.91	8.9	20.99	15.75
32	6.30	9.5	22.38	16.80
34	6.69	10.1	23.78	17.85
36	7.09	10.7	25.18	18.90
38	7.48	11.2	26.58	20.00
40	7.87	11.8	27.98	21.00
42	8.27	12.4	29.38	22.00
44	8.66	13.0	30.78	23.10
46	9.06	13.6	32.18	24.10
48	9.45	14.2	33.58	25.20
50	9.84	14.8	34.98	26.20
52	10.24	15.4	36.38	27.30
54	10.63	15.9	37.77	28.30
56	11.02	16.5	39.17	29.40
58	11.42	17.1	40.57	30.40
60	11.81	17.7	41.97	31.50
62	12.21	18.3	43.37	32.50
64	12.60	18.9	44.77	33.60
66	12.99	19.5	46.17	34.60
68	13.39	20.1	47.57	35.70

For more complete coverage, see *CRSI Manual of Standard Practice*.

Total weight = (wt. per ft x height) + F

F = weight to add for finishing

(this includes 1<sup>1</sup>/<sub>2</sub> turns at the top and 1<sup>1</sup>/<sub>2</sub> turns at the bottom of spiral)

For additional information see MnDOT 2472 and AASHTO LRFD 5.10.4.2

TENSION LAP SPLICES FOR EPOXY COATED BARS WITH >12" CONCRETE CAST BELOW

(Applies only to epoxy coated bars with an angle to the horizontal of 0 to 45 degrees)

$f_y=60$  ksi  $f_c=4$  ksi

Conc. Cover	Bar Size	Reinforcement Bar Spacing															
		4"		5"		5 1/2"		6"		6 1/2"		7"		7 1/2"		≥ 8"	
		Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B
2"	3	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"
	4	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	5	2'-7"	3'-4"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"
	6	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"
	7	3'-11"	5'-1"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"
	8	5'-2"	6'-8"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"
	9	6'-6"	8'-6"	5'-3"	6'-9"	5'-1"	6'-7"	5'-1"	6'-7"	5'-1"	6'-7"	5'-1"	6'-7"	5'-1"	6'-7"	5'-1"	6'-7"
10	8'-3"	10'-9"	6'-7"	8'-7"	6'-3"	8'-2"	6'-3"	8'-2"	6'-3"	8'-2"	6'-3"	8'-2"	6'-3"	8'-2"	6'-3"	8'-2"	
11	10'-2"	13'-3"	8'-2"	10'-7"	7'-6"	9'-9"	7'-6"	9'-9"	7'-6"	9'-9"	7'-6"	9'-9"	7'-6"	9'-9"	7'-6"	9'-9"	
14	N/A	N/A	11'-9"	15'-3"	10'-8"	13'-10"	10'-4"	13'-5"	10'-4"	13'-5"	10'-4"	13'-5"	10'-4"	13'-5"	10'-4"	13'-5"	
2 3/8"	3	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"
	4	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	5	2'-7"	3'-4"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"
	6	3'-1"	4'-0"	3'-1"	4'-0"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"
	7	3'-11"	5'-1"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"
	8	5'-2"	6'-8"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"
	9	6'-6"	8'-6"	5'-3"	6'-9"	4'-9"	6'-2"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"
10	8'-3"	10'-9"	6'-7"	8'-7"	6'-0"	7'-10"	5'-6"	7'-2"	5'-6"	7'-2"	5'-6"	7'-2"	5'-6"	7'-2"	5'-6"	7'-2"	
11	10'-2"	13'-3"	8'-2"	10'-7"	7'-5"	9'-8"	6'-10"	8'-10"	6'-8"	8'-7"	6'-8"	8'-7"	6'-8"	8'-7"	6'-8"	8'-7"	
14	N/A	N/A	11'-9"	15'-3"	10'-8"	13'-10"	9'-9"	12'-9"	9'-1"	11'-10"	9'-1"	11'-10"	9'-1"	11'-10"	9'-1"	11'-10"	
≥ 3"	3	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"
	4	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	5	2'-7"	3'-4"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"
	6	3'-1"	4'-0"	3'-1"	4'-0"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"	2'-10"	3'-8"
	7	3'-11"	5'-1"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-4"	4'-4"	3'-4"	4'-4"	3'-4"	4'-4"	3'-4"	4'-4"
	8	5'-2"	6'-8"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"
	9	6'-6"	8'-6"	5'-3"	6'-9"	4'-9"	6'-2"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"
10	8'-3"	10'-9"	6'-7"	8'-7"	6'-0"	7'-10"	5'-6"	7'-2"	5'-3"	6'-9"	5'-3"	6'-9"	5'-3"	6'-9"	5'-3"	6'-9"	
11	10'-2"	13'-3"	8'-2"	10'-7"	7'-5"	9'-8"	6'-10"	8'-10"	6'-3"	8'-2"	5'-10"	7'-7"	5'-10"	7'-6"	5'-10"	7'-6"	
14	N/A	N/A	11'-9"	15'-3"	10'-8"	13'-10"	9'-9"	12'-9"	9'-0"	11'-9"	8'-5"	10'-11"	7'-10"	10'-2"	7'-8"	9'-11"	

Table is based on AASHTO Article 5.10.8.2.1a. It includes modification factors for reinforcement location, bar coating, normal weight concrete, and reinforcement confinement per AASHTO Articles 5.10.8.2.1b and 5.10.8.2.1c. Reinforcement confinement is conservatively calculated by taking transverse reinforcement index as 0. Excess reinforcement factor is conservatively taken as 1.0. Tension lap splice lengths are based on AASHTO Article 5.10.8.4.3a. Concrete cover is defined as the cover to the bar being considered. For concrete cover or bar spacing that falls between table values, conservatively use lap length shown in the table for smaller concrete cover or bar spacing. The Class A splice length is equivalent to the rebar development length.

TENSION LAP SPLICES	Percent of $A_s$ spliced within required lap length	
	$A_{s, provided}/A_{s, required} \leq 2$	$A_{s, provided}/A_{s, required} > 2$
$A_{s, provided}/A_{s, required} \geq 2$	Class A	Class B
$A_{s, provided}/A_{s, required} < 2$	Class B	Class B

Where:  $A_{s, provided}$  = Area of reinforcement provided and  $A_{s, required}$  = Area of reinforcement required by analysis

Figure 5.2.2.1  
Reinforcement Data

TENSION LAP SPLICES FOR EPOXY COATED BARS WITH ≤ 12" CONCRETE CAST BELOW

(Applies to all other epoxy coated bars not covered by Figure 5.2.2.1)

$f_y=60$  ksi  $f_c=4$  ksi

Conc. Cover	Bar Size	Reinforcement Bar Spacing																
		4"		5"		5 1/2"		6"		6 1/2"		7"		7 1/2"		≥ 8"		
		Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	
1"	3	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	1'-5"	1'-10"	
	4	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	
	5	2'-9"	3'-6"	2'-9"	3'-6"	2'-9"	3'-6"	2'-9"	3'-6"	2'-9"	3'-6"	2'-9"	3'-6"	2'-9"	3'-6"	2'-9"	3'-6"	
	6	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	
	7	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	
	8	6'-0"	7'-10"	6'-0"	7'-10"	6'-0"	7'-10"	6'-0"	7'-10"	6'-0"	7'-10"	6'-0"	7'-10"	6'-0"	7'-10"	6'-0"	7'-10"	
	9	7'-4"	9'-7"	7'-4"	9'-7"	7'-4"	9'-7"	7'-4"	9'-7"	7'-4"	9'-7"	7'-4"	9'-7"	7'-4"	9'-7"	7'-4"	9'-7"	
	10	8'-11"	11'-7"	8'-11"	11'-7"	8'-11"	11'-7"	8'-11"	11'-7"	8'-11"	11'-7"	8'-11"	11'-7"	8'-11"	11'-7"	8'-11"	11'-7"	
	11	10'-6"	13'-8"	10'-6"	13'-8"	10'-6"	13'-8"	10'-6"	13'-8"	10'-6"	13'-8"	10'-6"	13'-8"	10'-6"	13'-8"	10'-6"	13'-8"	
	14	N/A	N/A	14'-0"	18'-2"	14'-0"	18'-2"	14'-0"	18'-2"	14'-0"	18'-2"	14'-0"	18'-2"	14'-0"	18'-2"	14'-0"	18'-2"	
	1 1/2"	3	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"
		4	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"
		5	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"
		6	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"
7		3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	3'-7"	4'-8"	
8		4'-6"	5'-11"	4'-6"	5'-11"	4'-6"	5'-11"	4'-6"	5'-11"	4'-6"	5'-11"	4'-6"	5'-11"	4'-6"	5'-11"	4'-6"	5'-11"	
9		5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	
10		7'-4"	9'-6"	6'-10"	8'-11"	6'-10"	8'-11"	6'-10"	8'-11"	6'-10"	8'-11"	6'-10"	8'-11"	6'-10"	8'-11"	6'-10"	8'-11"	
11		9'-0"	11'-8"	8'-2"	10'-7"	8'-2"	10'-7"	8'-2"	10'-7"	8'-2"	10'-7"	8'-2"	10'-7"	8'-2"	10'-7"	8'-2"	10'-7"	
14		N/A	N/A	11'-0"	14'-4"	11'-0"	14'-4"	11'-0"	14'-4"	11'-0"	14'-4"	11'-0"	14'-4"	11'-0"	14'-4"	11'-0"	14'-4"	
2"		3	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"
		4	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"
		5	2'-3"	3'-0"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"
		6	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"
	7	3'-6"	4'-6"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	
	8	4'-6"	5'-11"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	
	9	5'-9"	7'-6"	4'-7"	6'-0"	4'-6"	5'-10"	4'-6"	5'-10"	4'-6"	5'-10"	4'-6"	5'-10"	4'-6"	5'-10"	4'-6"	5'-10"	
	10	7'-4"	9'-6"	5'-10"	7'-7"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	
	11	9'-0"	11'-8"	7'-2"	9'-4"	6'-8"	8'-8"	6'-8"	8'-8"	6'-8"	8'-8"	6'-8"	8'-8"	6'-8"	8'-8"	6'-8"	8'-8"	
	14	N/A	N/A	10'-4"	13'-5"	9'-5"	12'-3"	9'-1"	11'-10"	9'-1"	11'-10"	9'-1"	11'-10"	9'-1"	11'-10"	9'-1"	11'-10"	
	2 3/8"	3	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"
		4	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"
		5	2'-3"	3'-0"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"
		6	2'-9"	3'-7"	2'-9"	3'-7"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"
7		3'-6"	4'-6"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	
8		4'-6"	5'-11"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	
9		5'-9"	7'-6"	4'-7"	6'-0"	4'-2"	5'-5"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	
10		7'-4"	9'-6"	5'-10"	7'-7"	5'-4"	6'-11"	4'-11"	6'-4"	4'-10"	6'-4"	4'-10"	6'-4"	4'-10"	6'-4"	4'-10"	6'-4"	
11		9'-0"	11'-8"	7'-2"	9'-4"	6'-7"	8'-6"	6'-0"	7'-10"	5'-10"	7'-7"	5'-10"	7'-7"	5'-10"	7'-7"	5'-10"	7'-7"	
14		N/A	N/A	10'-4"	13'-5"	9'-5"	12'-3"	8'-8"	11'-3"	8'-1"	10'-5"	8'-1"	10'-5"	8'-1"	10'-5"	8'-1"	10'-5"	
≥ 3"		3	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"	1'-1"	1'-5"
		4	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"
		5	2'-3"	3'-0"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"
		6	2'-9"	3'-7"	2'-9"	3'-7"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"	2'-2"	2'-10"
	7	3'-6"	4'-6"	3'-2"	4'-2"	3'-2"	4'-2"	3'-2"	4'-2"	2'-7"	3'-4"	2'-7"	3'-4"	2'-7"	3'-4"	2'-7"	3'-4"	
	8	4'-6"	5'-11"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	3'-8"	4'-9"	2'-11"	3'-9"	2'-11"	3'-9"	2'-11"	3'-9"	
	9	5'-9"	7'-6"	4'-7"	6'-0"	4'-2"	5'-5"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	4'-1"	5'-4"	
	10	7'-4"	9'-6"	5'-10"	7'-7"	5'-4"	6'-11"	4'-11"	6'-4"	4'-7"	6'-0"	4'-7"	6'-0"	4'-7"	6'-0"	4'-7"	6'-0"	
	11	9'-0"	11'-8"	7'-2"	9'-4"	6'-7"	8'-6"	6'-0"	7'-10"	5'-7"	7'-2"	5'-2"	6'-8"	5'-1"	6'-8"	5'-1"	6'-8"	
	14	N/A	N/A	10'-4"	13'-5"	9'-5"	12'-3"	8'-8"	11'-3"	8'-0"	10'-4"	7'-5"	9'-7"	6'-11"	9'-0"	6'-9"	8'-9"	

Table is based on AASHTO Article 5.10.8.2.1a. It includes modification factors for reinforcement location, bar coating, normal weight concrete, and reinforcement confinement per AASHTO Articles 5.10.8.2.1b and 5.10.8.2.1c. Reinforcement confinement is conservatively calculated by taking transverse reinforcement index as 0. Excess reinforcement factor is conservatively taken as 1.0. Tension lap splice lengths are based on AASHTO Article 5.10.8.4.3a. Concrete cover is defined as the cover to the bar being considered. For concrete cover or bar spacing that falls between table values, conservatively use lap length shown in the table for smaller concrete cover or bar spacing. The Class A splice length is equivalent to the rebar development length.

TENSION LAP SPLICES	Percent of $A_s$ spliced within required lap length	
	$A_{s, provided}/A_{s, required}$	
$\geq 2$	≤ 50	> 50
$< 2$	Class A	Class B
	Class B	Class B

Where:  $A_{s, provided}$  = Area of reinforcement provided and  $A_{s, required}$  = Area of reinforcement required by analysis

Figure 5.2.2  
Reinforcement Data

**TENSION LAP SPLICES FOR PLAIN UNCOATED BARS WITH >12" CONCRETE CAST BELOW**

(Applies only to plain uncoated bars with an angle to the horizontal of 0 to 45 degrees)

$f_y=60$  ksi       $f_c'=4$  ksi

Conc. Cover	Bar Size	Reinforcement Spacing															
		4"		5"		5 1/2"		6"		6 1/2"		7"		7 1/2"		≥ 8"	
		Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B
2"	3	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"
	4	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"
	5	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"
	6	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"
	7	3'-0"	3'-11"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"
	8	3'-11"	5'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"
	9	5'-0"	6'-6"	4'-0"	5'-2"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"
	10	6'-4"	8'-3"	5'-1"	6'-7"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"	4'-10"	6'-3"
≥ 3"	11	7'-10"	10'-1"	6'-3"	8'-1"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"	5'-9"	7'-6"
	14	N/A	N/A	9'-0"	11'-8"	8'-2"	10'-7"	7'-11"	10'-3"	7'-11"	10'-3"	7'-11"	10'-3"	7'-11"	10'-3"	7'-11"	10'-3"
	3	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"
	4	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"	1'-7"	2'-1"
	5	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"
	6	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"	2'-5"	3'-1"
	7	3'-0"	3'-11"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"
	8	3'-11"	5'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"	3'-2"	4'-1"
≥ 3"	9	5'-0"	6'-6"	4'-0"	5'-2"	3'-8"	4'-9"	3'-7"	4'-7"	3'-7"	4'-7"	3'-7"	4'-7"	3'-7"	4'-7"	3'-7"	4'-7"
	10	6'-4"	8'-3"	5'-1"	6'-7"	4'-7"	6'-0"	4'-3"	5'-6"	4'-0"	5'-2"	4'-0"	5'-2"	4'-0"	5'-2"	4'-0"	5'-2"
	11	7'-10"	10'-1"	6'-3"	8'-1"	5'-8"	7'-4"	5'-3"	6'-9"	4'-10"	6'-3"	4'-6"	5'-10"	4'-5"	5'-9"	4'-5"	5'-9"
	14	N/A	N/A	9'-0"	11'-8"	8'-2"	10'-7"	7'-6"	9'-9"	6'-11"	9'-0"	6'-5"	8'-4"	6'-0"	7'-10"	5'-10"	7'-7"

Table is based on AASHTO Article 5.10.8.2.1a. It includes modification factors for reinforcement location, bar coating, normal weight concrete, and reinforcement confinement per AASHTO Articles 5.10.8.2.1b and 5.10.8.2.1c. Reinforcement confinement is conservatively calculated by taking transverse reinforcement index as 0. Excess reinforcement factor is conservatively taken as 1.0. Tension lap splice lengths are based on AASHTO Article 5.10.8.4.3a. Concrete cover is defined as the cover to the bar being considered. For concrete cover or bar spacing that falls between table values, conservatively use lap length shown in the table for smaller concrete cover or bar spacing. The Class A splice length is equivalent to the rebar development length.

TENSION LAP SPLICES	Percent of $A_s$ spliced within required lap length	
$A_{s, provided}/A_{s, required}$	≤ 50	> 50
≥ 2	Class A	Class B
< 2	Class B	Class B

Where:  $A_{s, provided}$  = Area of reinforcement provided    and     $A_{s, required}$  = Area of reinforcement required by analysis

**Figure 5.2.2.3  
Reinforcement Data**

**TENSION LAP SPLICES FOR PLAIN UNCOATED BARS WITH ≤ 12" CONCRETE CAST BELOW**

(Applies to all other plain uncoated bars not covered by Figure 5.2.2.3)

$f_y=60$  ksi       $f'_c=4$  ksi

Conc. Cover	Bar Size	Reinforcement Spacing															
		4"		5"		5 1/2"		6"		6 1/2"		7"		7 1/2"		≥ 8"	
		Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B
2"	3	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"
	4	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"
	5	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	6	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"
	7	2'-4"	3'-0"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"
	8	3'-0"	3'-11"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"
	9	3'-10"	5'-0"	3'-1"	4'-0"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"
	10	4'-11"	6'-4"	3'-11"	5'-1"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"	3'-9"	4'-10"
≥ 3"	11	6'-0"	7'-10"	4'-10"	6'-3"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"
	14	N/A	N/A	6'-11"	9'-0"	6'-4"	8'-2"	6'-1"	7'-11"	6'-1"	7'-11"	6'-1"	7'-11"	6'-1"	7'-11"	6'-1"	7'-11"
	3	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"	1'-0"	1'-3"
	4	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"	1'-3"	1'-7"
	5	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	6	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"	1'-10"	2'-5"
	7	2'-4"	3'-0"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"	2'-2"	2'-9"
	8	3'-0"	3'-11"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"	2'-5"	3'-2"
9	3'-10"	5'-0"	3'-1"	4'-0"	2'-10"	3'-8"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	3'-7"	2'-9"	
10	4'-11"	6'-4"	3'-11"	5'-1"	3'-7"	4'-7"	3'-3"	4'-3"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	
11	6'-0"	7'-10"	4'-10"	6'-3"	4'-5"	5'-8"	4'-0"	5'-3"	3'-9"	4'-10"	3'-5"	4'-6"	3'-5"	4'-5"	3'-5"	4'-5"	
14	N/A	N/A	6'-11"	9'-0"	6'-4"	8'-2"	5'-9"	7'-6"	5'-4"	6'-11"	4'-11"	6'-5"	4'-8"	6'-0"	4'-6"	5'-10"	

Table is based on AASHTO Article 5.10.8.2.1a. It includes modification factors for reinforcement location, bar coating, normal weight concrete, and reinforcement confinement per AASHTO Articles 5.10.8.2.1b and 5.10.8.2.1c. Reinforcement confinement is conservatively calculated by taking transverse reinforcement index as 0. Excess reinforcement factor is conservatively taken as 1.0. Tension lap splice lengths are based on AASHTO Article 5.10.8.4.3a. Concrete cover is defined as the cover to the bar being considered. For concrete cover or bar spacing that falls between table values, conservatively use lap length shown in the table for smaller concrete cover or bar spacing. The Class A splice length is equivalent to the rebar development length.

TENSION LAP SPLICES	Percent of $A_s$ spliced within required lap length	
$A_{s, provided}/A_{s, required}$	≤ 50	> 50
≥ 2	Class A	Class B
< 2	Class B	Class B

Where:  $A_{s, provided}$  = Area of reinforcement provided and  $A_{s, required}$  = Area of reinforcement required by analysis

**Figure 5.2.2.4  
Reinforcement Data**

TENSION LAP SPLICES FOR STAINLESS STEEL BARS WITH >12" CONCRETE CAST BELOW

(Applies only to stainless steel bars with an angle to the horizontal of 0 to 45 degrees)

$f_y=75$  ksi  $f_c=4$  ksi

Conc. Cover	Bar Size	Reinforcement Bar Spacing																
		4"		5"		5 1/2"		6"		6 1/2"		7"		7 1/2"		≥ 8"		
		Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	
2"	3	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	
	4	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	
	5	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	
	6	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	
	7	3'-9"	4'-11"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	
	8	4'-11"	6'-5"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	
	9	6'-3"	8'-1"	5'-0"	6'-6"	4'-7"	5'-11"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	
	10	7'-11"	10'-3"	6'-4"	8'-3"	5'-9"	7'-6"	5'-3"	6'-10"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	
	11	9'-9"	12'-8"	7'-10"	10'-1"	7'-1"	9'-2"	6'-6"	8'-5"	6'-0"	7'-10"	5'-7"	7'-3"	5'-6"	7'-2"	5'-6"	7'-2"	
	14	N/A	N/A	11'-3"	14'-7"	10'-2"	13'-3"	9'-4"	12'-2"	8'-8"	11'-3"	8'-0"	10'-5"	7'-6"	9'-9"	7'-4"	9'-6"	
	2 3/8"	3	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"
		4	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"
		5	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"
		6	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"
7		3'-9"	4'-11"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	
8		4'-11"	6'-5"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	
9		6'-3"	8'-1"	5'-0"	6'-6"	4'-7"	5'-11"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	
10		7'-11"	10'-3"	6'-4"	8'-3"	5'-9"	7'-6"	5'-3"	6'-10"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	
11		9'-9"	12'-8"	7'-10"	10'-1"	7'-1"	9'-2"	6'-6"	8'-5"	6'-0"	7'-10"	5'-7"	7'-3"	5'-6"	7'-2"	5'-6"	7'-2"	
14		N/A	N/A	11'-3"	14'-7"	10'-2"	13'-3"	9'-4"	12'-2"	8'-9"	11'-4"	8'-9"	11'-4"	8'-9"	11'-4"	8'-9"	11'-4"	
≥ 3"		3	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"	1'-6"	1'-11"
		4	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"	2'-0"	2'-7"
		5	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"	2'-6"	3'-3"
		6	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"	3'-0"	3'-10"
	7	3'-9"	4'-11"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	3'-5"	4'-6"	
	8	4'-11"	6'-5"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	3'-11"	5'-1"	
	9	6'-3"	8'-1"	5'-0"	6'-6"	4'-7"	5'-11"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	4'-5"	5'-9"	
	10	7'-11"	10'-3"	6'-4"	8'-3"	5'-9"	7'-6"	5'-3"	6'-10"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	
	11	9'-9"	12'-8"	7'-10"	10'-1"	7'-1"	9'-2"	6'-6"	8'-5"	6'-0"	7'-10"	5'-7"	7'-3"	5'-6"	7'-2"	5'-6"	7'-2"	
	14	N/A	N/A	11'-3"	14'-7"	10'-2"	13'-3"	9'-4"	12'-2"	8'-8"	11'-3"	8'-0"	10'-5"	7'-6"	9'-9"	7'-4"	9'-6"	

Table is based on AASHTO Article 5.10.8.2.1a. It includes modification factors for reinforcement location, bar coating, normal weight concrete, and reinforcement confinement per AASHTO Articles 5.10.8.2.1b and 5.10.8.2.1c. Reinforcement confinement is conservatively calculated by taking transverse reinforcement index as 0. Excess reinforcement factor is conservatively taken as 1.0. Tension lap splice lengths are based on AASHTO Article 5.10.8.4.3a. Concrete cover is defined as the cover to the bar being considered. For concrete cover or bar spacing that falls between table values, conservatively use lap length shown in the table for smaller concrete cover or bar spacing. The Class A splice length is equivalent to the rebar development length.

TENSION LAP SPLICES	Percent of $A_s$ spliced within required lap length	
	$A_{s, provided}/A_{s, required} \leq 50$	$A_{s, provided}/A_{s, required} > 50$
$\geq 2$	Class A	Class B
$< 2$	Class B	Class B

Where:  $A_{s, provided}$  = Area of reinforcement provided and  $A_{s, required}$  = Area of reinforcement required by analysis

Figure 5.2.2.5  
Reinforcement Data



TENSION LAP SPLICES FOR STAINLESS STEEL BARS WITH ≤ 12" CONCRETE CAST BELOW

(Applies to all other stainless steel bars not covered by Figure 5.2.2.5)

$f_y=75$  ksi  $f_c=4$  ksi

Conc. Cover	Bar Size	Reinforcement Bar Spacing															
		4"		5"		5 1/2"		6"		6 1/2"		7"		7 1/2"		≥ 8"	
		Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B	Class A	Class B
1"	3	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"
	4	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	5	2'-3"	2'-11"	2'-3"	2'-11"	2'-3"	2'-11"	2'-3"	2'-11"	2'-3"	2'-11"	2'-3"	2'-11"	2'-3"	2'-11"	2'-3"	2'-11"
	6	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"	3'-1"	4'-0"
	7	4'-0"	5'-3"	4'-0"	5'-3"	4'-0"	5'-3"	4'-0"	5'-3"	4'-0"	5'-3"	4'-0"	5'-3"	4'-0"	5'-3"	4'-0"	5'-3"
	8	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"	5'-0"	6'-6"
	9	6'-2"	8'-0"	6'-2"	8'-0"	6'-2"	8'-0"	6'-2"	8'-0"	6'-2"	8'-0"	6'-2"	8'-0"	6'-2"	8'-0"	6'-2"	8'-0"
10	7'-5"	9'-8"	7'-5"	9'-8"	7'-5"	9'-8"	7'-5"	9'-8"	7'-5"	9'-8"	7'-5"	9'-8"	7'-5"	9'-8"	7'-5"	9'-8"	
11	8'-9"	11'-5"	8'-9"	11'-5"	8'-9"	11'-5"	8'-9"	11'-5"	8'-9"	11'-5"	8'-9"	11'-5"	8'-9"	11'-5"	8'-9"	11'-5"	
14	N/A	N/A	11'-8"	15'-2"	11'-8"	15'-2"	11'-8"	15'-2"	11'-8"	15'-2"	11'-8"	15'-2"	11'-8"	15'-2"	11'-8"	15'-2"	
1 1/2"	3	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"
	4	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	5	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	6	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"
	7	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"
	8	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"
	9	4'-10"	6'-3"	4'-8"	6'-1"	4'-8"	6'-1"	4'-8"	6'-1"	4'-8"	6'-1"	4'-8"	6'-1"	4'-8"	6'-1"	4'-8"	6'-1"
10	6'-1"	7'-11"	5'-8"	7'-5"	5'-8"	7'-5"	5'-8"	7'-5"	5'-8"	7'-5"	5'-8"	7'-5"	5'-8"	7'-5"	5'-8"	7'-5"	
11	7'-6"	9'-9"	6'-10"	8'-10"	6'-10"	8'-10"	6'-10"	8'-10"	6'-10"	8'-10"	6'-10"	8'-10"	6'-10"	8'-10"	6'-10"	8'-10"	
14	N/A	N/A	9'-2"	11'-11"	9'-2"	11'-11"	9'-2"	11'-11"	9'-2"	11'-11"	9'-2"	11'-11"	9'-2"	11'-11"	9'-2"	11'-11"	
2"	3	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"
	4	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	5	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	6	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"
	7	2'-11"	3'-9"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"
	8	3'-9"	4'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"
	9	4'-10"	6'-3"	3'-10"	5'-0"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"	3'-9"	4'-11"
10	6'-1"	7'-11"	4'-11"	6'-4"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	4'-8"	6'-0"	
11	7'-6"	9'-9"	6'-0"	7'-10"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	5'-7"	7'-2"	
14	N/A	N/A	8'-8"	11'-3"	7'-10"	10'-2"	7'-7"	9'-10"	7'-7"	9'-10"	7'-7"	9'-10"	7'-7"	9'-10"	7'-7"	9'-10"	
2 3/8"	3	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"
	4	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	5	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	6	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"
	7	2'-11"	3'-9"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"
	8	3'-9"	4'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"
	9	4'-10"	6'-3"	3'-10"	5'-0"	3'-6"	4'-7"	3'-5"	4'-5"	3'-5"	4'-5"	3'-5"	4'-5"	3'-5"	4'-5"	3'-5"	4'-5"
10	6'-1"	7'-11"	4'-11"	6'-4"	4'-5"	5'-9"	4'-1"	5'-3"	4'-1"	5'-3"	4'-1"	5'-3"	4'-1"	5'-3"	4'-1"	5'-3"	
11	7'-6"	9'-9"	6'-0"	7'-10"	5'-6"	7'-1"	5'-0"	6'-6"	4'-11"	6'-4"	4'-11"	6'-4"	4'-11"	6'-4"	4'-11"	6'-4"	
14	N/A	N/A	8'-8"	11'-3"	7'-10"	10'-2"	7'-2"	9'-4"	6'-9"	8'-9"	6'-9"	8'-9"	6'-9"	8'-9"	6'-9"	8'-9"	
≥ 3"	3	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"	1'-2"	1'-6"
	4	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"	1'-6"	2'-0"
	5	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"	1'-11"	2'-6"
	6	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"	2'-3"	3'-0"
	7	2'-11"	3'-9"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"	2'-8"	3'-5"
	8	3'-9"	4'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"	3'-0"	3'-11"
	9	4'-10"	6'-3"	3'-10"	5'-0"	3'-6"	4'-7"	3'-5"	4'-5"	3'-5"	4'-5"	3'-5"	4'-5"	3'-5"	4'-5"	3'-5"	4'-5"
10	6'-1"	7'-11"	4'-11"	6'-4"	4'-5"	5'-9"	4'-1"	5'-3"	3'-10"	5'-0"	3'-10"	5'-0"	3'-10"	5'-0"	3'-10"	5'-0"	
11	7'-6"	9'-9"	6'-0"	7'-10"	5'-6"	7'-1"	5'-0"	6'-6"	4'-8"	6'-0"	4'-4"	5'-7"	4'-3"	5'-6"	4'-3"	5'-6"	
14	N/A	N/A	8'-8"	11'-3"	7'-10"	10'-2"	7'-2"	9'-4"	6'-8"	8'-8"	6'-2"	8'-0"	5'-9"	7'-6"	5'-8"	7'-4"	

Table is based on AASHTO Article 5.10.8.2.1a. It includes modification factors for reinforcement location, bar coating, normal weight concrete, and reinforcement confinement per AASHTO Articles 5.10.8.2.1b and 5.10.8.2.1c. Reinforcement confinement is conservatively calculated by taking transverse reinforcement index as 0. Excess reinforcement factor is conservatively taken as 1.0. Tension lap splice lengths are based on AASHTO Article 5.10.8.4.3a. Concrete cover is defined as the cover to the bar being considered. For concrete cover or bar spacing that falls between table values, conservatively use lap length shown in the table for smaller concrete cover or bar spacing. The Class A splice length is equivalent to the rebar development length.

TENSION LAP SPLICES	Percent of $A_s$ spliced within required lap length	
$A_{s, provided}/A_{s, required}$	≤ 50	> 50
≥ 2	Class A	Class B
< 2	Class B	Class B

Where:  $A_{s, provided}$  = Area of reinforcement provided and  $A_{s, required}$  = Area of reinforcement required by analysis

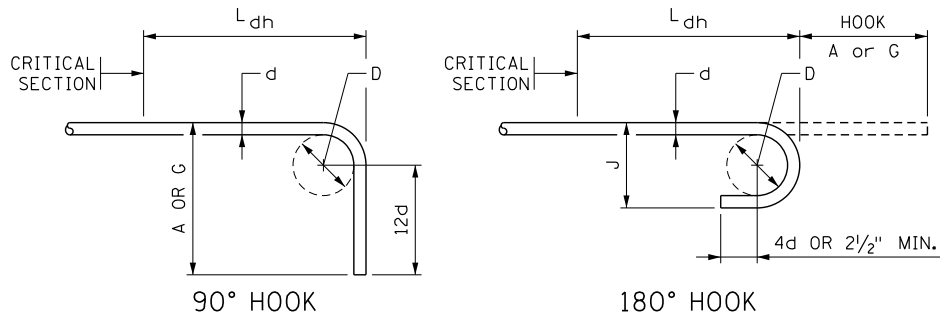
Figure 5.2.2.6  
Reinforcement Data

**DEVELOPMENT LENGTH FOR STANDARD HOOKS IN TENSION**

For plain and epoxy bars,  $f_y = 60$  ksi  $f'_c = 4$ ksi  
 For stainless steel bars,  $f_y = 75$  ksi

<b>Hooked Bar Development Length, <math>L_{dh}</math></b> When: Side Cover $\geq 2.5"$ AND For 90° and 180° Hooks, Concrete Cover $\geq 2"$ in the direction of bar extension				<b>Hooked Bar Development Length, <math>L_{dh}</math></b> When: Side Cover $< 2.5"$ OR For 90° and 180° Hooks, Concrete Cover $< 2"$ in the direction of bar extension			
Bar Size	Plain Uncoated Bars	Epoxy Coated Bars	Stainless Steel Bars	Bar Size	Plain Uncoated Bars	Epoxy Coated Bars	Stainless Steel Bars
3	6"	7"	8"	3	8"	9"	9"
4	8"	10"	10"	4	10"	1'-0"	1'-0"
5	10"	1'-0"	1'-0"	5	1'-0"	1'-3"	1'-3"
6	1'-0"	1'-2"	1'-3"	6	1'-3"	1'-6"	1'-6"
7	1'-2"	1'-4"	1'-5"	7	1'-5"	1'-8"	1'-9"
8	1'-4"	1'-7"	1'-7"	8	1'-7"	1'-11"	2'-0"
9	1'-6"	1'-9"	1'-10"	9	1'-10"	2'-2"	2'-3"
10	1'-8"	2'-0"	2'-1"	10	2'-1"	2'-5"	2'-7"
11	1'-10"	2'-2"	2'-3"	11	2'-3"	2'-9"	2'-10"
14	2'-9"	3'-3"	3'-5"	14	2'-9"	3'-3"	3'-5"

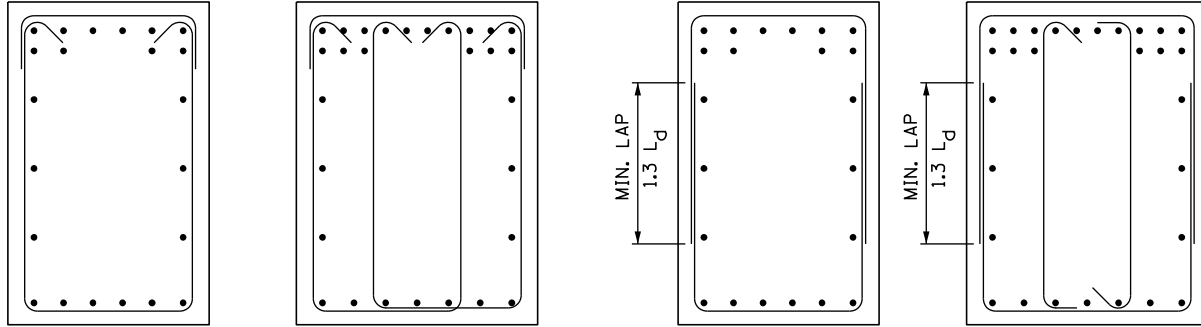
Table is based on AASHTO Article 5.10.8.2.4a and includes modification factors per LRFD Art. 5.10.8.2.4b for normal weight concrete, bar coating, and reinforcement confinement. The reinforcement confinement factor is not applicable to bars larger than No. 11 bars. Note that MnDOT allows use of No. 14 bar standard hooks for concrete strengths up to 10 ksi.



BAR SIZE	D	180° HOOKS		90° HOOKS
		A OR G	J	A OR G
3	2 1/4"	5"	3"	6"
4	3"	6"	4"	8"
5	3 3/4"	7"	5"	10"
6	4 1/2"	8"	6"	1'-0"
7	5 1/4"	10"	7"	1'-2"
8	6"	11"	8"	1'-4"
9	9/2"	1'-3"	11 3/4"	1'-7"
10	10 3/4"	1'-5"	1'-1 1/4"	1'-10"
11	12"	1'-7"	1'-2 3/4"	2'-0"
14	18 1/4"	2'-3"	1'-9 3/4"	2'-7"

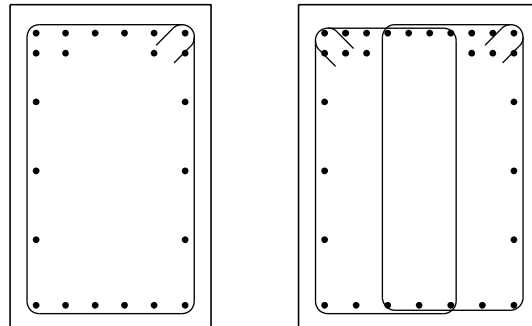
STANDARD HOOK DIMENSIONS FOR BARS IN TENSION

**Figure 5.2.2.7**  
**Reinforcement Data**



PREFERRED METHOD

OTHER ACCEPTABLE METHODS



REQUIRED METHOD TO RESIST TORSION

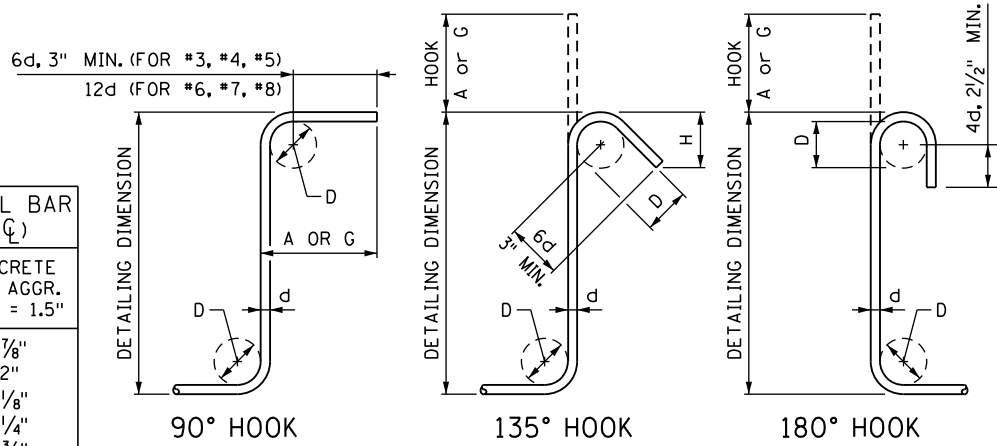
METHODS FOR ANCHORAGE OF SHEAR REINFORCEMENT

SEE AASHTO LRFD 5.10.8.2.6b AND 5.10.8.2.6d

MINIMUM HORIZONTAL BAR SPACING (C <sub>c</sub> TO C <sub>c</sub> )		
BAR SIZE	CONCRETE MAX AGGR. SIZE = 1.0"	CONCRETE MAX AGGR. SIZE = 1.5"
3	2 5/8"	1 7/8"
4	2 3/4"	2"
5	2 7/8"	2 1/8"
6	3"	2 1/4"
7	3 1/8"	2 3/8"
8	3 1/4"	2 1/2"
9	3 1/2"	2 7/8"
10	3 5/8"	3 1/4"
11	3 3/4"	3 5/8"
14	4 1/4"	4 1/4"

SEE AASHTO LRFD 5.10.3.1.1

MINIMUM SPACING FOR PARALLEL BARS IN A LAYER



90° HOOK

135° HOOK

180° HOOK

BAR SIZE	D	90° HOOKS		135° HOOKS		180° HOOKS	
		A OR G	A OR G	H	A OR G	H	
3	2"	4 1/2"	4 1/2"	2 3/4"	5"	2 3/4"	
4	2 1/2"	4 3/4"	5"	3"	5 1/2"	3 1/2"	
5	3 1/4"	6"	6"	3 3/4"	6 1/2"	4 1/2"	
6	4 1/2"	1'-0"	8"	4 1/2"	8 1/4"	6"	
7	5 1/4"	1'-2"	9"	5 1/4"	9 3/4"	7"	
8	6"	1'-4"	10 1/2"	6"	11"	8"	

DIMENSIONS AND DETAILS BASED ON CRSI MANUAL OF STANDARD PRACTICE  
STANDARD STIRRUP AND TIE HOOK DIMENSIONS

Figure 5.2.2.8  
Reinforcement Data

### **5.2.3 General Reinforcement Practices**

Reinforcement practices follow those shown by the Concrete Reinforcing Steel Institute (CRSI) in the *Manual of Standard Practice*. These practices include:

- 1) For bent bars, omit the last length dimension on reinforcement bar details.
- 2) Use standard length bars for all but the last bar in long bar runs.
- 3) Use a maximum length of 60 feet for #4 deck or slab bars and 40 feet for other applications.
- 4) Use a maximum length of 60 feet for bars #5 and larger.
- 5) Recognize that bar cutting and bending tolerances are  $\pm 1$  inch for bars and that this tolerance is important for long straight bars that do not have lap splices to provide dimensional flexibility. Refer to MnDOT document *Suggested Reinforcement Detailing Practices*, which is located at <http://www.dot.state.mn.us/bridge/drafting-aids.html>, for more guidance on rebar detailing to account for tolerances.
- 6) Reinforcement bars longer than 60 feet or larger than #11 are available only on special order, and should be avoided. Designers should check with the State Bridge Design Engineer before using special order sizes or lengths.

### **5.2.4 Reinforcement Bar Couplers**

Reinforcement bar couplers are expensive compared to conventional lap splices. Where lap splices cannot be readily used (bridge widening projects, staged construction, large river pier longitudinal bars—anywhere that the available space for a rebar projection is limited), couplers should be considered. Where possible, stagger reinforcement bar couplers in order to distribute the stiffness of the couplers. There are numerous coupler types and sizes. For members that require couplers, consider the coupler outside diameter and length when detailing reinforcement, in order to avoid congestion and clear cover issues.

### **5.2.5 Adhesive Anchors**

Similar to bar couplers, adhesive anchors are expensive. Adhesive anchors are typically used to attach secondary structural members to new concrete or primary structural members to existing (old) concrete. A typical use is to attach a metal rail to a concrete base.

See Article 13.3.2 of this manual for an adhesive anchor design example.

Refer to Technical Memorandum No. 18-11-B-01 for use of adhesive anchors in sustained tension applications.

**5.2.6 Shrinkage and Temperature Reinforcement [5.10.6]**

Follow the requirements for shrinkage and temperature reinforcement given in LRFD Article 5.10.6. An exception to this is that shrinkage and temperature reinforcement is not required in buried footings of typical bridges.

**5.3 Concrete Slabs**

In many bridge engineering documents the terms “concrete slab” and “concrete deck” are used interchangeably. Within this manual, “concrete slab” will refer to a superstructure type without supporting beam elements. In most cases, the primary reinforcement for slabs is parallel to the centerline of roadway. Likewise, within this manual “concrete decks” will refer to the superstructure element placed on top of beams or girders. In most cases, the primary reinforcement for a concrete deck is transverse to the centerline of roadway. Practices for concrete decks are described in Section 9 of this manual.

**5.3.1 Geometry**

The maximum span lengths for concrete slabs are as follows:

Number of Spans	Without Haunches	With Haunches
1	30 ft	40 ft
2	40 ft	50 ft
3 or 4	50 ft	60 ft

End spans should be approximately 80% of the center span length to balance moments and prevent uplift.

LRFD Table 2.5.2.6.3-1 provides guidance for recommended minimum structure depth as a function of span length for slab superstructures without haunches.

When haunches are required, use linear haunches in accordance with the following:

$$\text{Minimum slab depth at pier} = 1.33 \cdot \left[ \frac{S + 10}{30} \right]$$

(includes wear course if present)

$$\text{Minimum slab depth in non-haunched area} = 0.8 \cdot \left[ \frac{S + 10}{30} \right]$$

(includes wear course if present)

$$\text{Minimum haunch length } L = 0.15 \cdot S$$

(where S is the length of longest span)

**5.3.2****Design/Analysis**

Requirements for skewed slab type bridges are as follows:

- Skew can be ignored for slab bridges with skew angles of 20° or less. Place transverse reinforcement parallel to substructures.
- For slab bridges with skew angles between 20° and 45°, perform a two-dimensional plate analysis. Place transverse reinforcement normal to the bridge centerline.
- Slab type bridges are not allowed for bridges with skew angles greater than 45°.

Slab bridges curved in plan may be designed as if straight. Designers should consider and investigate the need for providing additional reinforcement in the portion of the slab outside of chord lines connecting substructure units.

Do not include the concrete wearing course in section properties when performing strength and serviceability checks. This will ensure that the slab has adequate capacity if traffic is carried on the bridge during operations associated with milling off the old wearing course and the placement of a new wearing course. An exception to this is when checking the top reinforcement in the negative moment region for flexural crack control.

**[5.6.7]**

When checking crack control for slabs and decks, use the Class 2 exposure condition ( $\gamma_e = 0.75$ ). Although the actual clear cover may exceed 2 inches for the slab/deck top bars, calculate  $d_c$  using a maximum clear concrete cover equal to 2 inches.

Determine reinforcement bar cutoff points based on strength, serviceability, and minimum reinforcement requirements.

**[5.12.2.1]**

Although not required by AASHTO, MnDOT requires a check of one-way shear in slab bridges. For determination of the live load distribution factor for shear, assume that the live load is distributed over the entire width of the superstructure. Load all lanes and use the appropriate multiple presence factor. For determination of factored shear resistance, use  $\beta = 2.0$ . If shear reinforcement is needed, try thickening the slab to eliminate it. If shear reinforcement must be used, calculate the appropriate  $\beta$  and  $\sigma$  values using LRFD Article 5.7.3.4.2.

**[5.12.2.1]**

For determination of the bottom transverse distribution reinforcement, apply the percentage required by LRFD Article 5.12.2.1 to the interior strip reinforcement. Using the exterior strip as the basis for the distribution reinforcement is not necessary because:

- The interior strip sees the majority of traffic.

- The exterior strip contains a larger amount of reinforcement in order to stiffen the unsupported edge.
- The exterior strip has additional support/stiffness provided by the barrier/parapet/curb.

### **5.3.3 Exterior Strip [4.6.2.1.4b]**

Outside edges of slab bridges contain the exterior strip or edge beam. At a minimum, the exterior strip reinforcement must match that of the interior portions of the bridge.

Special consideration for the design of edge beams is required for bridges with sidewalks. Separately poured sidewalks may be considered to act compositely with the slab when adequate means of shear transfer at the interface is provided.

### **5.3.4 Reinforcement Layout**

Use the following guidelines for layout of reinforcement in a simple span slab bridge (see example in Figure 5.3.4.1):

Interior strip reinforcement

- Top longitudinal – 1 spacing, 1 bar size
- Bottom longitudinal – 2 spacings, 1 bar size

Exterior strip reinforcement

- Top longitudinal – 1 spacing, 1 bar size
  - Bottom longitudinal – 2 spacings, 1 bar size
- Transverse reinforcement – 1 spacing, 1 bar size

Use the following guidelines for layout of reinforcement in a continuous slab bridge:

Option 1 (see example in Figure 5.3.4.2):

Interior strip reinforcement

- Top longitudinal – 2 spacings, 1 bar size
- Bottom longitudinal – 2 spacings, 1 bar size

Exterior strip reinforcement

- Top longitudinal – 2 spacings, 1 bar size
  - Bottom longitudinal – 2 spacings, 1 bar size
- Transverse reinforcement – 1 spacing, 1 bar size

Option 2 (see example in Figure 5.3.4.3):

Interior strip reinforcement

- Top longitudinal – 2 spacings, 2 bar sizes
- Bottom longitudinal – 2 spacings, 2 bar sizes

Exterior strip reinforcement

- Top longitudinal – 2 spacings, 2 bar sizes
  - Bottom longitudinal – 2 spacings, 2 bar sizes
- Transverse reinforcement - 1 spacing, 1 bar size

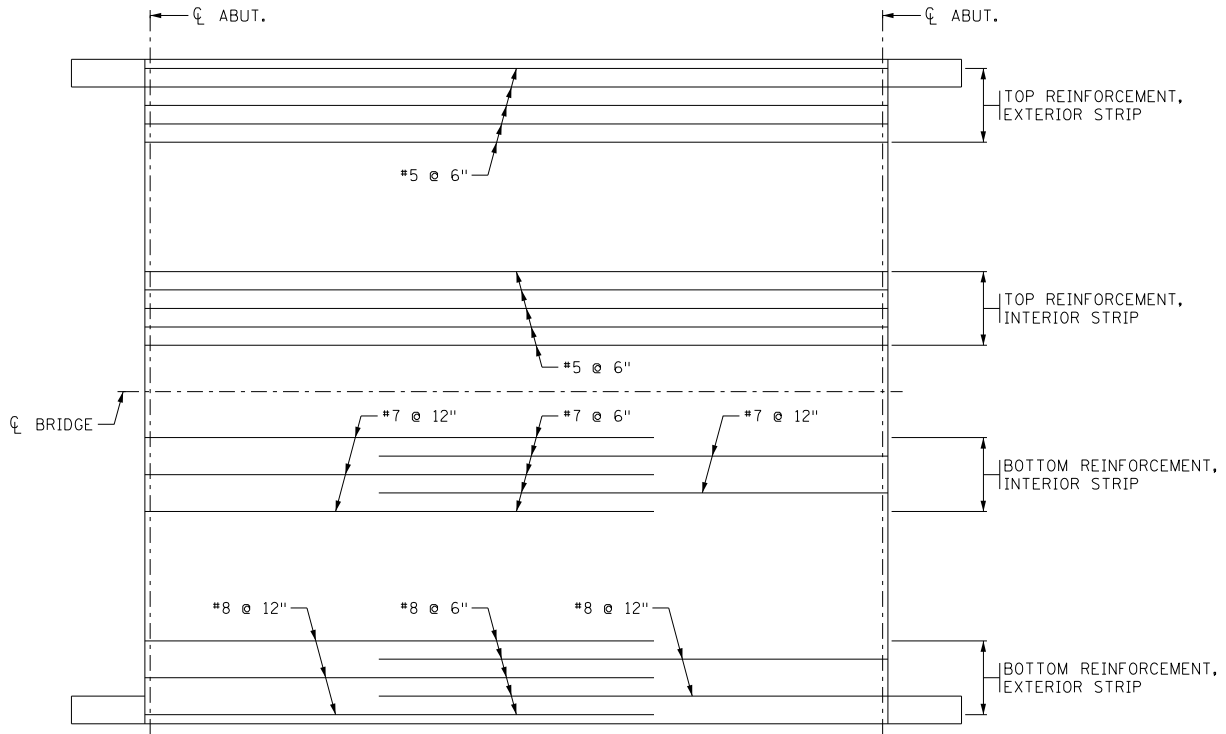


Figure 5.3.4.1

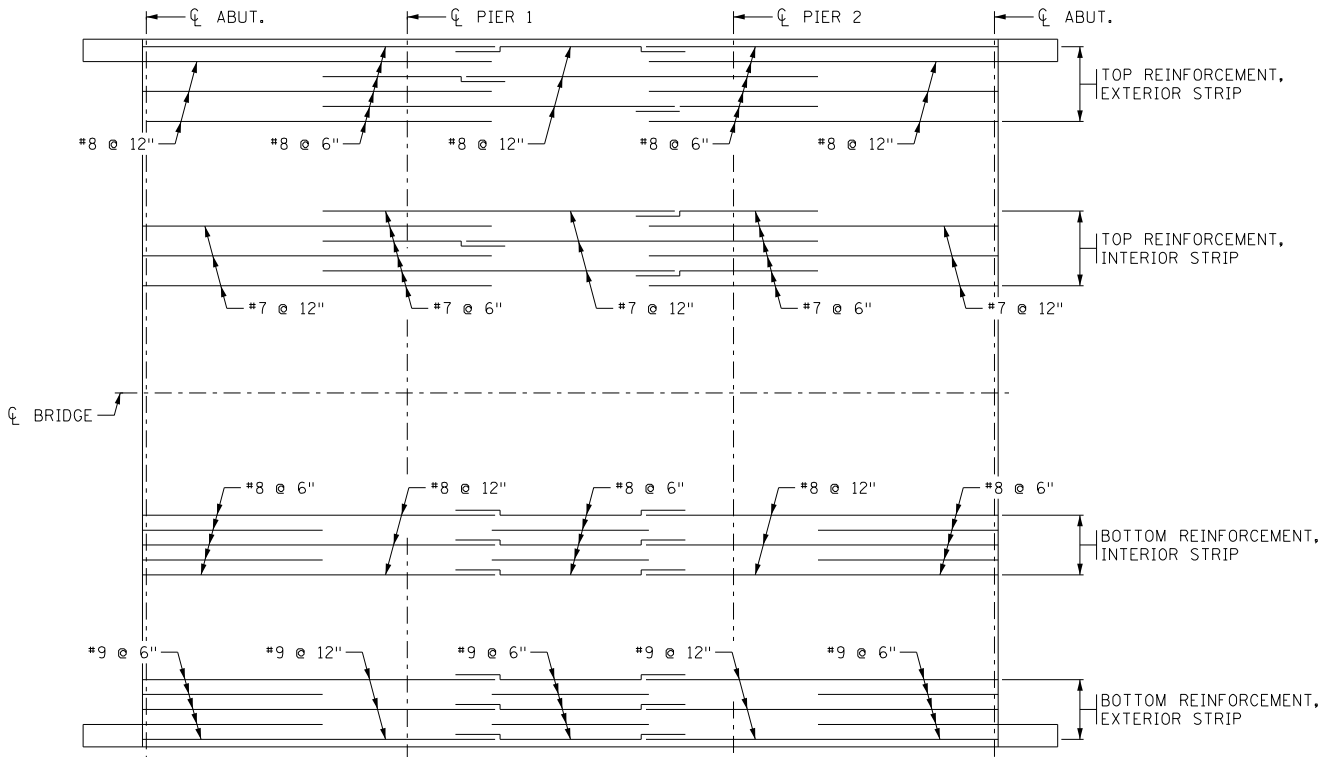
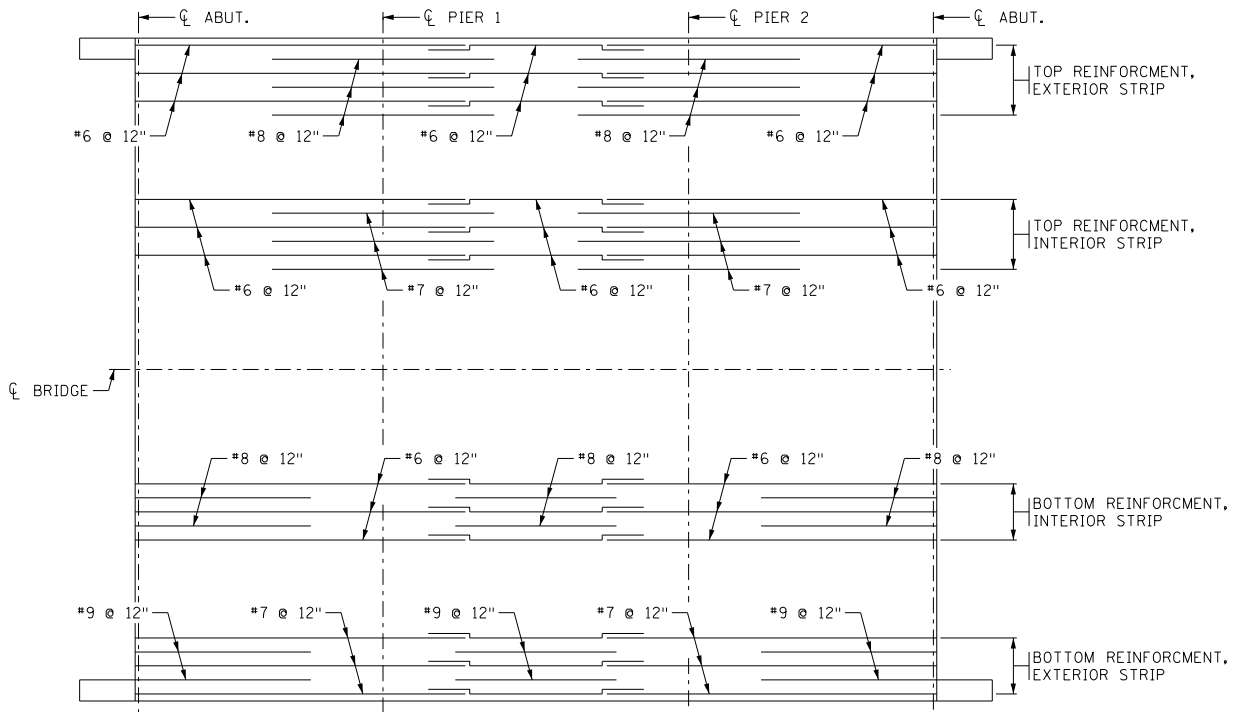


Figure 5.3.4.2





**Figure 5.3.4.3**

To simplify placement, detail reinforcement such that top bars are positioned over bottom bars where possible. For example, if the design requires bottom longitudinal bars spaced on 10 inch centers, top longitudinal bars might be spaced on 10 inch centers in positive moment regions and 5 inch centers in negative moment regions.

Extend railing dowel bars to the bottom layer of slab reinforcement and provide a horizontal leg for ease of placement.

**5.3.5 Camber and Deflections**  
**[5.6.3.5.2]**

In order to obtain the best rideability over the life of the structure, camber concrete slab bridges for the immediate dead load deflection plus one half of the long-term deflection. Use gross section properties for dead load deflection calculations and a long-term creep multiplier of 4.0.

Railings, sidewalks, medians, and wearing courses are not placed while the slab is supported on falsework. Assume that only the slab carries the dead load of these elements.

Check live load deflections using the effective moment of inertia. The effective moment of inertia may be approximated as one half of the gross moment of inertia. The maximum live load deflection is  $L/800$  for vehicular bridges that do not carry pedestrians and  $L/1000$  for vehicular bridges that carry pedestrians.

Consider the concrete wearing course to be functioning compositely with the slab for live load deflection. Assume the riding surface has lost  $1/2$  inch of thickness due to wear.

Use a live load distribution factor equal to the number of lanes times the multiple presence factor and divide by the width of the slab for the deflection check.

#### **5.4 Pretensioned Concrete**

The details of pretensioned concrete beams are presented on standard *Bridge Details Part II* sheets incorporated into a set of plans. Prepare a separate sheet for each type of beam in the project. Beams are identical if they have the same cross-section, strand layout, concrete strengths, and a similar length. To simplify fabrication and construction, try to minimize the number of beam types incorporated into a project. Design exterior beams with a strength equal to or greater than the interior beams.

##### **5.4.1 Geometry**

Provide a minimum stool along centerline of beam that is based on  $1\frac{1}{2}$  inches of minimum stool at edge of flange. For initial dead load computations, a stool height equal to the minimum stool height plus 1 inch may be assumed. For final design of the beam, assume a stool dead load based on the average of the calculated minimum and maximum stool along the beam centerline. Deck cross slopes, horizontal curves, and vertical curves all impact the stool height. For determination of composite beam properties, use a stool height equal to  $1\frac{1}{2}$  inches.

There are several Bridge Office practices regarding the type and location of diaphragms or cross frames for prestressed beam bridges:

- 1) Design prestressed I-beam bridges without continuity over the piers, except in the following situations:
  - a) Bridge is over water with pile bent piers supported by unstable soils such as fat clay.
  - b) Bridge is over water with pile bent piers at risk for large ice or debris loading and pier does not have an encasement wall.
- 2) Intermediate diaphragms are not required for 14RB, 18RB, 22RB, 27M, 30MH, and 35MH beams. For all other standard beam sizes, the following applies. Intermediate diaphragms are not required for single

spans of 45'-0" or less. Provide one diaphragm per every 45 feet of span length, spaced evenly along the span as stated in Table 5.4.1.1.

**Table 5.4.1.1**

Span length (ft)	Base number of intermediate diaphragms
Less than 45'-0"	0
45'-0" to 90'-0"	1 located at midspan
90'-0" to 135'-0"	2 located at the third points
135'-0" to 180'-0"	3 located at the quarter points
Greater than 180'-0"	4 plus an additional diaphragm for each additional 45 ft of span length greater than 180'-0"

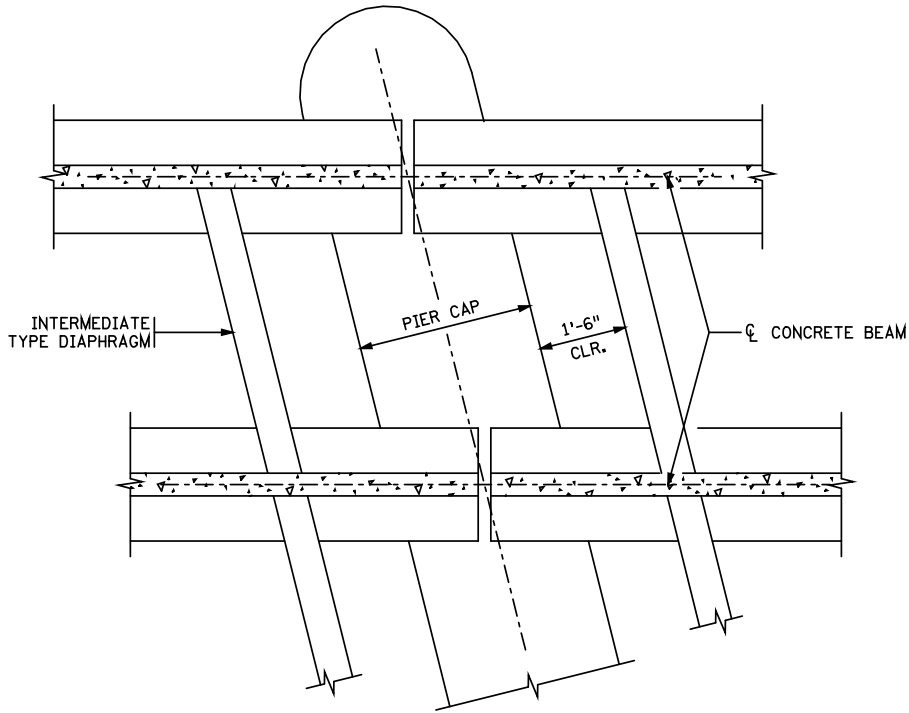
Do not place additional diaphragms in the fascia bay to brace against over-height traffic impact. From 1999 to 2018, standard practice was to include additional diaphragms in the fascia bay of spans over traffic, but impacted bridges have shown less damage without the diaphragm added.

- 3) Figure 5.4.1.1 illustrates the typical layout of intermediate diaphragms at piers for bridges without continuity over the piers.

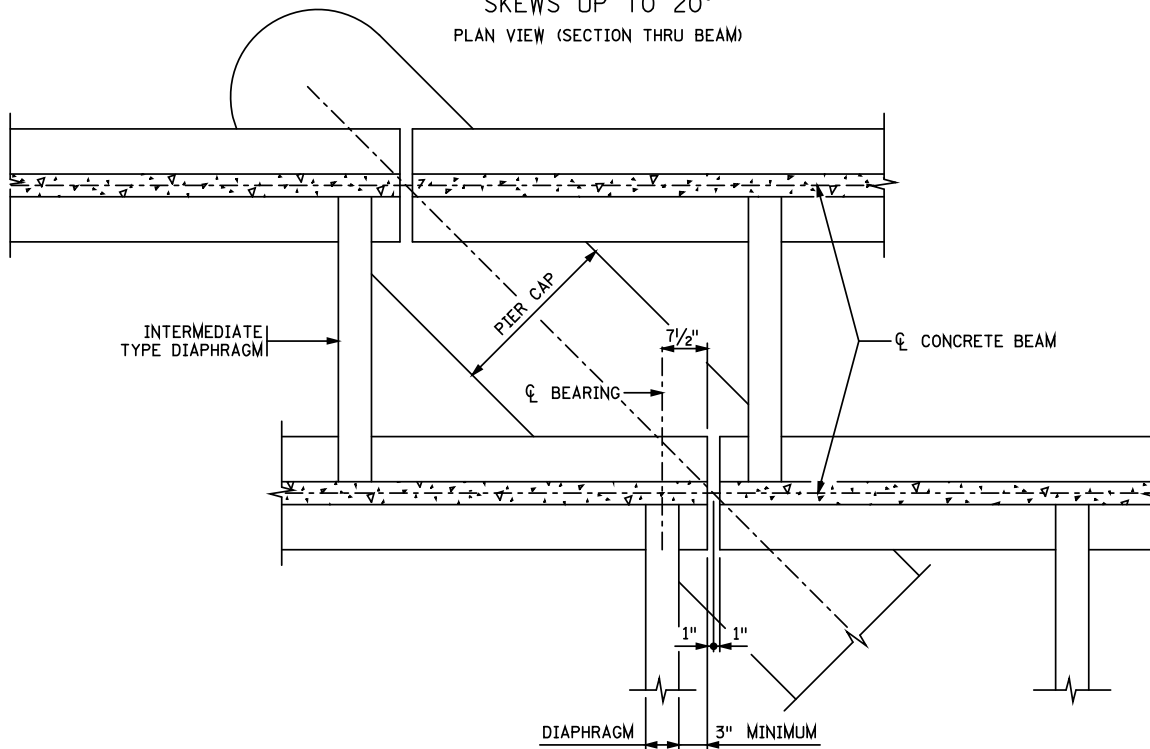
Locate the centerline of bearing  $7\frac{1}{2}$  inches from the end of the beam for RB, M, MH, and MN shapes. Locate the centerline of bearing  $8\frac{1}{2}$  inches from the end of the beam for MW shapes. For MW shapes, this dimension can be adjusted if used with higher movement bearings, as opposed to the typical curved plate elastomeric bearings shown in Section 14 of this manual. However, if the  $8\frac{1}{2}$  inch dimension is exceeded, a special design for the bearing, sole plate, and beam end region must be completed.

At piers of two span bridges, provide 2 inches of clearance between the ends of RB, M, MH, and MN beams. Provide 3 inches clearance for structures with three or more spans. Provide 4 inches of clearance between the ends of MW beams regardless of the number of spans. Note that the fabrication length tolerance for pretensioned I-beams is  $\pm\frac{1}{8}$ " per 10 feet of length. It may be necessary to cope beam flanges at piers for bridges with tight horizontal curves or at skewed abutments.

For bridges on significant grades ( $> 3\%$ ) the sloped length of the beam will be significantly longer than the horizontal length between substructure units. If the sloped length is  $\frac{1}{2}$  inch or more than the horizontal length, identify the sloped length dimension on the beam detail plan sheets.



SKEWS UP TO 20°  
PLAN VIEW (SECTION THRU BEAM)



SKEW OVER 20°  
PLAN VIEW (SECTION THRU BEAM)

**Figure 5.4.1.1**  
**Typical Diaphragm Layout at Piers for Prestressed Concrete Beam Bridge**  
**With Continuous Deck Over Piers**

For bridges with curved decks supported by straight pretensioned beams, use the following guidance when laying out the geometry:

- For exterior beam on outside of curve, provide a minimum overhang of 6 inches between the edge of deck and outside edge of beam flange at the beam ends.
- For exterior beam on inside of curve, provide a minimum overhang of 6 inches between the edge of deck and outside edge of beam flange at the beam midspan.
- When choosing the spacing from the exterior beam to the first interior beam, consider the capacity of the exterior beam. The curvature will likely cause load demands that differ from beams that have a constant overhang.
- Orient each of the first interior beams parallel to the corresponding exterior beam.
- If the angle between the exterior beams is small, consider making them parallel. Note that this may require rechecking the overhang geometry to meet the minimum overhang stated above.
- Space the rest of the beams as evenly as possible. It is preferable to use only one flared space, if possible, to limit the number of different beam lengths and diaphragm lengths.
- Use a minimum beam spacing of 4 feet for RB, M, and MN series beams to provide sufficient space for diaphragms and inspection. Use a minimum beam spacing of 5 feet for MH and MW beams.

#### **5.4.2 Stress Limits** **[5.9.2.2] [5.9.2.3]**

For typical prestressed beams, check tension and compression service load stresses at two stages. The first stage is when the prestress force is transferred to the beams in the fabricator's yard. The second stage is after all losses have occurred and the beam is in the fully constructed bridge.

Design pretensioned beams with a maximum tension at transfer (after initial losses) of:

- Locations not considering bonded reinforcement

$$f_{\text{init\_allow}} = 0.0948 \cdot \sqrt{f'_{ci}} \leq 0.2 \text{ ksi} \quad (\text{where } f'_{ci} \text{ is in ksi})$$

- Locations considering bonded reinforcement

$$f_{\text{init\_allow}} = 0.24 \cdot \sqrt{f'_{ci}} \quad (\text{where } f'_{ci} \text{ is in ksi})$$

When using the bonded reinforcement tension limit for simply supported pretensioned beams, provide a minimum area of developed longitudinal tension reinforcement in accordance with Table 5.4.2.1 unless calculated per AASHTO Article C5.9.2.3.1b.

**Table 5.4.2.1**  
**Minimum Top Flange Longitudinal Bonded Reinforcement at**  
**Beam Ends**

Beam Shape	Standard Plans $A_s$ ① (in <sup>2</sup> )	Minimum Required $A_s$ ② (in <sup>2</sup> )
M	3.2	2.6
MH	3.2	3.1
MN	4.7	3.8
MW	6.3	5.8

① Area of top flange mild reinforcement in beam end region included in Bridge Details Part II Fig. 5-397.501 through 5-397.532.

② Minimum required area of top flange mild reinforcement to develop the maximum tensile force permitted by the bonded reinforcement tension limit ( $f_{init\_allow}$ ) with a concrete strength ( $f'_c$ ) of 8 ksi or less.

Design pretensioned beams with a maximum tension after all losses of:

$$f_{final\_allow} = 0.19 \cdot \sqrt{f'_c} \quad (\text{where } f'_c \text{ is in ksi})$$

Determine live load distribution using the approximate methods of LRFD Article 4.6.2.2 and check tension stress after all losses using the Service III Limit State.

**[ This Page Intentionally Left Blank ]**

**5.4.3****Design/Analysis****[5.9.3.2.3a]****[5.9.3.3]**

Use gross beam properties for design (i.e - do not transform prestressing strands to get transformed properties). Calculate the instantaneous losses (elastic shortening losses) using LRFD Equation C5.9.3.2.3a-1. Also, do not include any elastic gains caused by application of loads. Use the "approximate method" given in LRFD Article 5.9.3.3 to compute time-dependent prestress losses.

Design all pretensioned beams using uncoated low relaxation 0.6 inch diameter strands ( $A_s = 0.217 \text{ in}^2$ ) and epoxy coated mild reinforcement.

At the time of prestress transfer (initial), the minimum required concrete strength ( $f'_{ci}$ ) is 4.5 ksi and the maximum is limited to 8.0 ksi. At the termination of the curing period (final), the minimum concrete strength ( $f'_c$ ) is 5 ksi and the maximum strength is 9.5 ksi. Higher initial or final strengths may only be used with approval from the State Bridge Design Engineer. Note that an initial concrete strength greater than 7.5 ksi may add cost to the beam. The fabricator cannot remove the beam from the bed until a cylinder break indicates the concrete has reached its specified initial strength. Strengths higher than 7.5 ksi may require the fabricator to leave the beam in the bed longer than the normal 16-18 hours or require increased amounts of superplastizer and cement, thereby increasing the cost of the beam.

Generally, fabricators have stated that it is most cost effective to design beams with concrete strengths up to 7.5 ksi initial and 9.5 ksi final with as few strands as possible. If the design requires a higher initial concrete strength, the initial concrete strength can be increased up to 8.0 ksi. When a designer is faced with the decision to add strands or increase the concrete strength, it is more economical to increase the concrete strength up to the maximum limits allowed.

Do not use the maximum concrete strengths listed above unless needed. Optimize the design by back-calculating the initial and final concrete strengths needed to meet the allowable stress limits, and then reanalyze the beam with the new values. Reanalysis is needed because changes to the concrete strengths  $f'_{ci}$  and  $f'_c$  affect the concrete modulus, which affects the prestress losses and the composite beam section modulus. Use the lowest concrete strengths needed for the design. Design the concrete strengths to the nearest 0.1 ksi and report these values on the standard beam sheet.

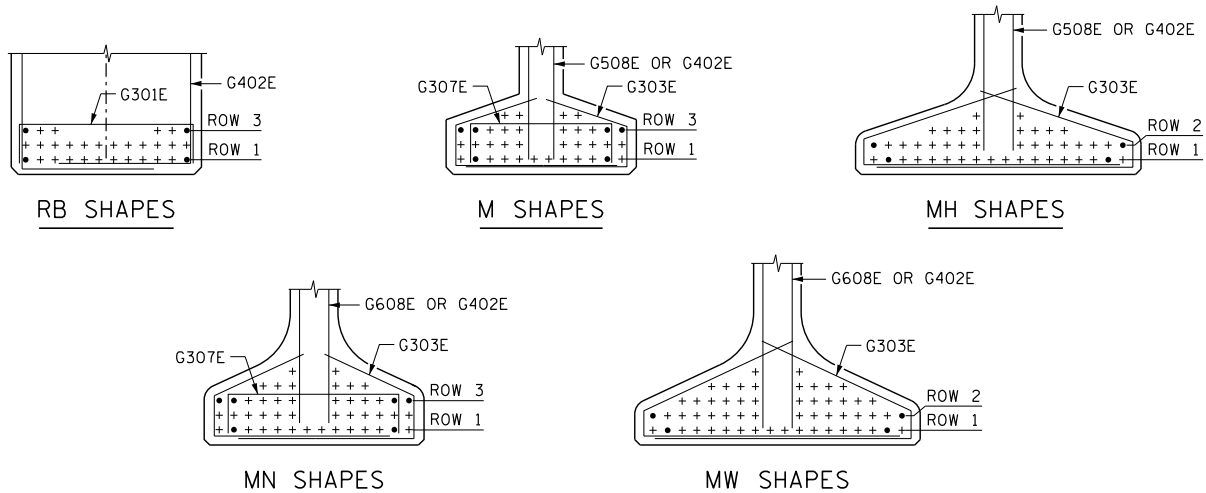
If possible, design so that the initial concrete strength is 0.5 to 1.0 ksi lower than the final concrete strength. Since concrete naturally gains



strength with age, the final strength of the beam will be more efficiently utilized.

In order to minimize the number of fabricator requests for strand pattern changes, use the following rules for placement of prestressing strands:

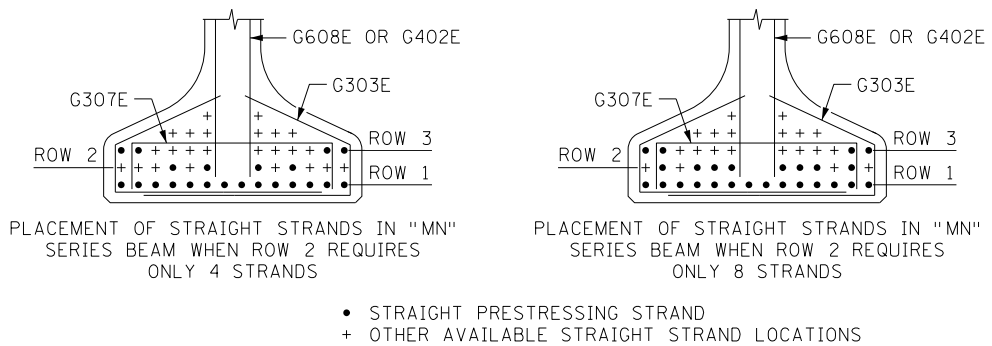
- Arrange straight strands in a 2 inch grid pattern with the bottom row of straight strands located 2 inches from the bottom of the beam. See standard beam sheets for possible strand locations.
- Use draped strands to reduce the initial required strength  $f'_{ci}$  at the end of the beam. Arrange draped strands in a 2 inch grid pattern independent of the straight strands. Locate draped strands starting 4 inches minimum from the bottom of the beam at the hold-downs and 3 inches minimum from the top at the end of the beam. Straight strands should be used in place of draped strands whenever possible. A design with the lowest number of draped strands is often preferred for economic and safety reasons. If using draped strands is necessary, it is desirable to minimize the number of draped strands and corresponding hold-down force. If possible, limit the draped strand angle of inclination to less than 6 degrees.
- For all designs, include a base set of straight strands in the locations shown in Figure 5.4.3.1. These base strands provide the fabricator a stable place to tie the flange confinement reinforcement, which in turn will be used to secure the stirrups in the bottom of the beam. For designs where fully tensioning all the base strands is undesirable, it is acceptable to pull selected pairs of the base strands to a lesser initial tension of 10 kips. If no draped strands are used, include a pair of straight strands in the web columns 6 inches from the bottom of the beam.



- BASE SET OF STRAIGHT STRANDS TO BE USED IN ALL BEAM DESIGNS. EACH BASE STRAND TO BE PULLED TO EITHER  $0.75 f_{pu}$   $A_{strand}$  OR 10 KIPS.
- + AVAILABLE STRAIGHT STRAND LOCATIONS TO BE USED AS NEEDED FOR BEAM DESIGN.

**Figure 5.4.3.1**

- After inclusion of the base set of strands, typically add other straight strands by starting from the bottom and moving up (i.e. – fill all of Row 1 and then all of Row 2, etc.) to get the largest eccentricity and therefore the most efficient design at midspan. Note that a smaller strand eccentricity is sometimes necessary, which may result in designs where the bottom rows are not entirely filled.
- For rows that do not need to be completely filled, fill rows from the inside out. For a given row, place the first straight strand in the column immediately outside of the stirrup, move two columns outward and place the next strand. Repeat until you reach the end of the row. If more strands are needed in the row, return to the first inside vacant column and again fill the rows from the inside out. See Figure 5.4.3.2 for examples. Place the strands with the goal of providing an approximately uniform prestress force across the width of the bottom flange.



STRAIGHT STRAND PLACEMENT EXAMPLES

**Figure 5.4.3.2**

Whenever possible, use a constant strand pattern for all girders on the same project. If the strand pattern varies between beams, the fabricator may be required to tension an entire bed length of strand in order to cast a single girder. This results in a large amount of wasted strand and will increase the cost of the beam.

For pretensioned I-beams with draped strands, the maximum total initial pretensioning force allowed is 3000 kips. This limit is based on the capacity of the fabricator pretensioning beds.

The maximum number of draped strands allowed at each hold-down point varies with the fabricator. Therefore, design and detail beams with one hold-down on each side of midspan, placed at 0.40L to 0.45L from the centerline of bearing. The fabricator will provide additional hold-downs as needed.

The following guidance is provided to designers to evaluate initial and final concrete service stresses to optimize their designs:

### **Final Concrete Stresses**

#### Midpoint Concrete Tension Stress at Bottom of Beam

If the Service III concrete tension stress is greater than the maximum allowable stress,  $f_{\text{final\_allow}} = 0.19 \cdot \sqrt{f'_c}$  (0.586 ksi for 9.5 ksi concrete):

- 1) Add 2 strands to the lowest row of straight strands or draped strands, whichever provides the lowest overall strand center of gravity at midspan.
- 2) Continue to add strands as stated above. If the tension stress is still greater than  $f_{\text{final\_allow}}$ , consider adding another line of beams to the bridge.

If the Service III concrete tension stress is less than  $f_{\text{final\_allow}}$ , two strands (either straight or draped) may be removed or the final concrete strength may be decreased and the beam reanalyzed. If the stress becomes greater than  $f_{\text{final\_allow}}$ , return to the original number of strands.

### **Initial Concrete Stresses**

#### Midpoint Concrete Compression Stress at Bottom of Beam

If the required initial concrete strength,  $f'_{ci}$ , is greater than 7.5 ksi:

- 1) Move the center of gravity of the strands up at midpoint of the beam until either the required final concrete strength becomes 9.5 ksi, or the required initial concrete strength is at or below 7.5 ksi and is 0.5 to 1.0 ksi lower than the final strength.
- 2) Remove 2 strands (preferably draped) from the beam and reanalyze. Keep in mind that changes will affect the required final concrete strength. If the removing of strands increases the final concrete strength above 9.5 ksi, return to the original number of strands.
- 3) If the guidance above still results in a required initial concrete strength greater than 7.5 ksi, the initial concrete strength may be increased up to a maximum value of 8.0 ksi. Note that this may increase the beam cost.

#### End Concrete Compression Stress at Bottom of Beam

If the required initial concrete strength,  $f'_{ci}$ , is greater than 7.5 ksi:

- 1) If the required initial concrete strength is lower than that calculated at the midpoint, draped strands may be replaced with straight strands. Note that these changes may affect the mid-beam stresses.
- 2) If the required initial concrete strength is greater than that calculated at the midpoint, the initial concrete strength may be increased up to a maximum value of 8.0 ksi. If 8.0 ksi is not

enough, some straight strands may be replaced with draped strands to decrease the required concrete strength. Note that changes to strand locations at the end of the beam may affect the midpoint stresses.

#### End Concrete Tension Stress at Top of Beam

If tension requires an initial concrete strength greater than 8.0 ksi, raise the center of gravity of the strands at the end of the beam. This can be accomplished by draping strands that were previously straight or increasing the height of the draped strands.

#### Midpoint Concrete Tension Stress at Top of Beam

If tension requires an initial concrete strength greater than that calculated at the bottom end or midpoint:

- 1) The center of gravity of the strands may be moved higher at the midpoint to decrease the required initial concrete strength.
- 2) The number of strands may be reduced to decrease the required initial concrete strength.

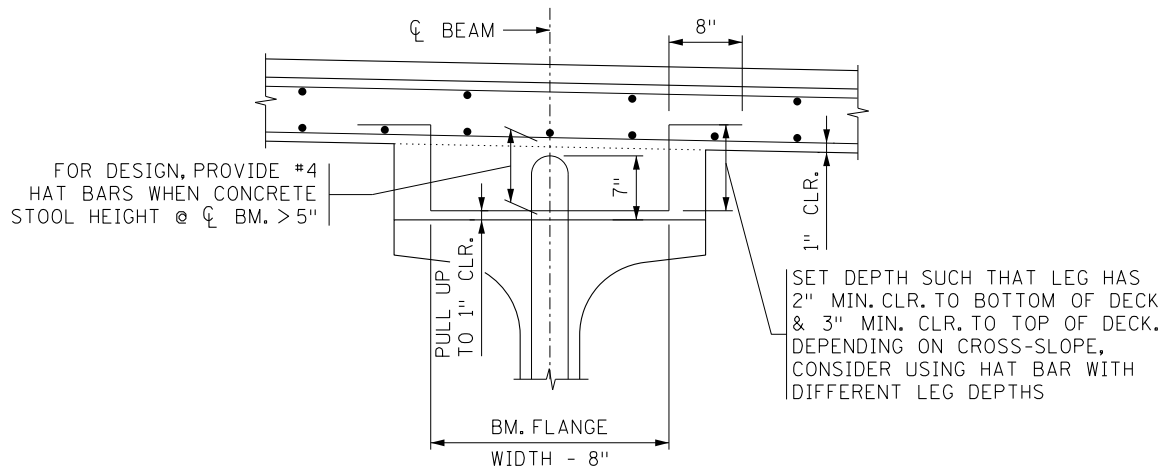
#### **[5.9.4.4.1]**

Ensure that adequate shear and splitting reinforcement is provided in the ends of beams. For RB, M, and MH series beams, the maximum size for stirrup bars is #5. For MN and MW series beams, the maximum size for stirrup bars is #6. In order to achieve proper concrete consolidation, the minimum spacing for #5 stirrups is  $2\frac{1}{2}$  inches and the minimum spacing for #6 stirrups is 3 inches. If the required amount of splitting reinforcement cannot be provided within  $\frac{h}{4}$  of the end of the beam, provide the remainder at minimum spacing.

Design shear reinforcement using the "General Procedure" provisions given in LRFD Article 5.7.3.4.2.

Horizontal shear reinforcement must be provided to ensure composite action between the beam and deck. MnDOT standard beam sheets accomplish this by extending the beam stirrups into the deck (G402E & G508E with 7" projection for RB, M, MH, MN shapes and G404E & G508E with 7¼" projection for MW shapes). In order to ensure composite action, the shear reinforcement must extend into the deck far enough to engage the deck bottom mat of reinforcement. Check the stool heights over the length of the beams. For regions where stool heights are found greater than 5 inches at beam centerline, do not increase the stirrup length or pull up the stirrups, but rather provide #4 "hat" shaped bars as shown in Figure 5.4.3.2. Set the leg depth to provide 2 inches minimum clear to the bottom of deck and 3 inches clear to the top of deck for the upper hook, and 1 inch clear from the bottom of the bar to the beam flange. In cases where field personnel report excessive stools not anticipated in the bridge plan, discuss

with them whether one "hat" bar or two "Z" bars would be better for rebar placement.



#### HAT BAR FOR BEAMS WITH LARGE STOOL HEIGHTS

**Figure 5.4.3.3**

Due to the height of the "MW" series beams, investigate whether a deck pour sequence is needed to reduce the effects of beam end rotation on the end region of the deck.

For bridges with curved decks supported by straight pretensioned beams, use the following guidance for design:

- When determining load distribution and composite beam section properties for the exterior beam, use the cross-section at the 1/3 points along the beam.
- For design of flared beams with varying beam spacing, use the cross-section at the 2/3 point from the narrow end of the flare for calculation of load distribution, and the cross-section at the 1/3 point from the narrow end for calculation of composite beam section properties.

For new bridges with decks consisting of a partial depth deck with a 2 inch wearing course, conservatively design pretensioned beams assuming the entire section of partial deck plus wearing course is placed in a single pour instead of the actual two pours.

#### **5.4.4 Detailing/ Reinforcement**

Identify the beam type on the beam sheet by depth in inches and length rounded to the next highest foot. In the superstructure quantities list, identify the beam type by depth. For example, an MN45 beam, 72'-4" long would be "MN45-73" on the beam sheet and "MN45" in the quantities list.

Group beams of similar lengths with the same strand pattern into one type on a beam sheet. The pay item quantity will be the total length of beams (of each height) in feet.

On the framing plan, show the beam and diaphragm spacing, staging, type of diaphragms, centerline of piers, centerline of abutment and pier bearings, working points, beam marks (B1, B2 etc.), the "X" end of beams, and the type and location of bearings. One end of each beam is labeled the "X" end after fabrication. This is used during erection to ensure that the beams are properly placed. Many times diaphragm inserts are not symmetric and beams can only be placed one way.

#### **5.4.5 Camber and Deflection**

The standard beam sheets contain a camber diagram where designers are to provide camber information. Knowing the deflection values associated with prestressing and different dead load components, camber values can be obtained.

Use MnDOT camber multipliers when designing standard "M" series, "MH" series, and "MN" series I-beams. The MnDOT camber multipliers are used to approximately convert the prestress and selfweight deflections at the time of prestress transfer to the deflections at the time of erection. Use a camber multiplier of 1.40 for the prestress deflection component. Use a camber multiplier of 1.40 for the selfweight of the member. No multiplier is used for diaphragm dead loads, deck and stool dead loads or parapet and median dead loads. These camber multipliers differ from the PCI multipliers as they are based on research specific to MnDOT beams. They are based on a time lapse of 30 to 180 days between the time of prestress transfer and the time of beam erection for deck placement.

Use of the MnDOT camber multipliers does not apply to the "MW" series beams. Complete a refined camber analysis using an appropriate creep model for "MW" series camber determination. Then report the estimated camber values for various girder ages in the bridge plan.

The "Erection Camber" is the camber of the beam at the time of erection after the diaphragms are in place. The "Est. Dead Load Deflection" is the sum of deflections associated with the placement of the deck, railings, sidewalks, and stool. Do not include the weight of the future wearing surface when computing the dead load deflection.

If the resulting "Est. Residual Camber" is less than 1 inch, consider increasing the number of strands.

**5.4.6 Standard  
I-Beams**

I-beam sections available for use in Minnesota include the "M" series, "MH" series, "MN" series, and "MW" series. They range in depth from 27 inches to 96 inches. The available I-beam sections have changed over the years due to development of higher strength concretes and more efficient beam shapes.

The "M" series beams were developed to be more efficient than the standard AASHTO shapes developed in the 1950s. The "MN" series beams were developed in the mid-2000s to be more efficient than the "M" series by using wider top and bottom flanges and a 6<sup>1/2</sup> inch thick web. The "MW" series sections were developed in 2011 to allow for more efficiency and longer spans in the range of 150 to 200 feet. Upon creation of the "MN" and "MW" series, most of the M shapes (45M, 54M, 63M, 72M, and 81M) were archived. The 27M and 36M shapes continued to be available as there were no corresponding MN shape at those depths. In 2018, the "MH" series beams were developed to provide more efficient designs in the 75 to 115 foot span range. Figures 5.4.6.1 through 5.4.6.4 contain section properties and preliminary beam spacing vs. span length curves for all MnDOT standard I-beam shapes.

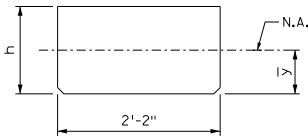
For beam span lengths greater than 145 ft, confirm with fabricators that the beams can be safely transported to the bridge project site.

**[5.9.4.4.2]**

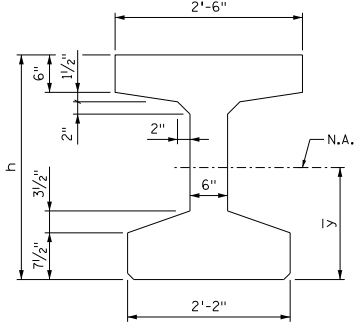
The G303E and G307E bars are bottom flange straight strand confinement bars. These bars are spaced with the stirrups and have a maximum spacing of 6 inches over 1.5 x (beam height) at each beam end.

**5.4.7 Standard  
Rectangular Beams**

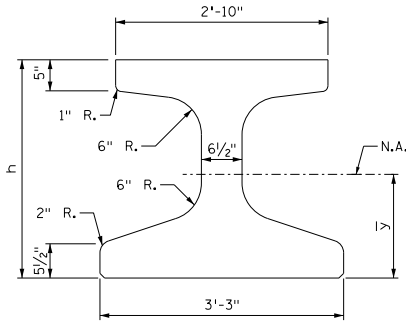
Solid rectangular pretensioned beams ("RB" series) may be used on short span bridges. These units are most appropriate for short span structures requiring a low profile or where construction of falsework for a slab structure would be difficult or unwanted. Figure 5.4.6.1 and 5.4.6.2 contain section properties and preliminary beam spacing vs. span length curves for the standard rectangular beams.



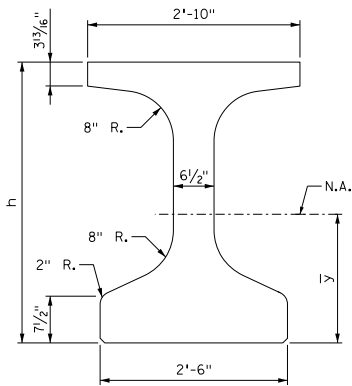
RECTANGULAR BEAM



18" M" SERIES I-BEAM



18" MH" SERIES I-BEAM



18" MN" SERIES I-BEAM

DESIGN ASSUMPTIONS FOR PRETENSIONED CONCRETE BEAM CHART:

2020 AASHTO LRFD Bridge Design Specifications, 9th Edition.

HL-93 Live Load

Beam Concrete:  $f'_c = 9.5 \text{ ksi}$   $f'_{ci} = 8.0 \text{ ksi}$   $w_{bm} = 0.155 \text{ kips/ft}^3$

$$E_c = 1265\sqrt{f'_c} + 1000 \text{ ksi}$$

Deck Concrete:  $f'_c = 4.0 \text{ ksi}$   $E_c = 3987 \text{ ksi}$

$w_c = 0.145 \text{ kcf}$  for  $E_c$  computation

$w_c = 0.150 \text{ kcf}$  for dead load computation

0.6" diameter low relaxation strands,  $E_s = 28,500 \text{ ksi}$

$f_{pu} = 300 \text{ ksi}$  with initial pull of  $0.72f_{pu}$

Simple supports with six beams and deck without wearing course.

Deck carries two Type S barriers with no sidewalk or median.

Skew = 0 degrees.

Deck thickness for dead load calculations based on BDM Table 9.2.1.1.

Effective deck thickness used for composite beam section properties is equal to total deck thickness minus  $1/2"$  of wear.

$2 1/2"$  average stool height used for dead load calculations.

$1 1/2"$  stool height used for composite beam section properties.

Barrier dead load is applied equally to all beams.

Dead load includes 0.020 ksf future wearing course.

Approximate long term losses are used per AASHTO LRFD Art. 5.9.5.3.

Service Concrete Tensile Stress Limits:

After Initial Losses:  $0.094\sqrt{f'_{ci}} \leq 0.2 \text{ ksi}$

After All Losses:  $0.19\sqrt{f'_c}$

Beam Properties

BEAM TYPE	h (in)	AREA (in <sup>2</sup> )	W ① (lb/ft)	$\bar{y}$ (in)	I (in <sup>4</sup> )	S <sub>b</sub> (in <sup>3</sup> )	A <sub>ct</sub> ② (in <sup>2</sup> )
14RB	14	364	392	7.00	5,945	849	312
18RB	18	468	504	9.00	12,640	1,404	364
22RB	22	572	616	11.00	23,070	2,097	416
27M	27	516	555	13.59	43,080	3,170	296
30MH	30	639	688	13.66	70,416	5,155	403
35MH	35	672	723	15.85	105,570	6,661	419
36M	36	570	614	17.96	93,530	5,208	323
40MH	40	704	758	18.07	149,002	8,246	435
MN45	45	690	743	20.58	178,780	8,687	427
MN54	54	749	806	24.63	285,230	11,580	457
MN63	63	807	869	28.74	421,750	14,670	486

① Based on 155 pounds per cubic foot.

② Based on a 9" slab with  $1/2"$  of wear and  $1 1/2"$  stool. See AASHTO LRFD Art. 5.7.3.4.2 for A<sub>ct</sub> definition.

**Figure 5.4.6.1**  
**Precast Prestressed Concrete Beam Data (RB, M, MH, MN)**



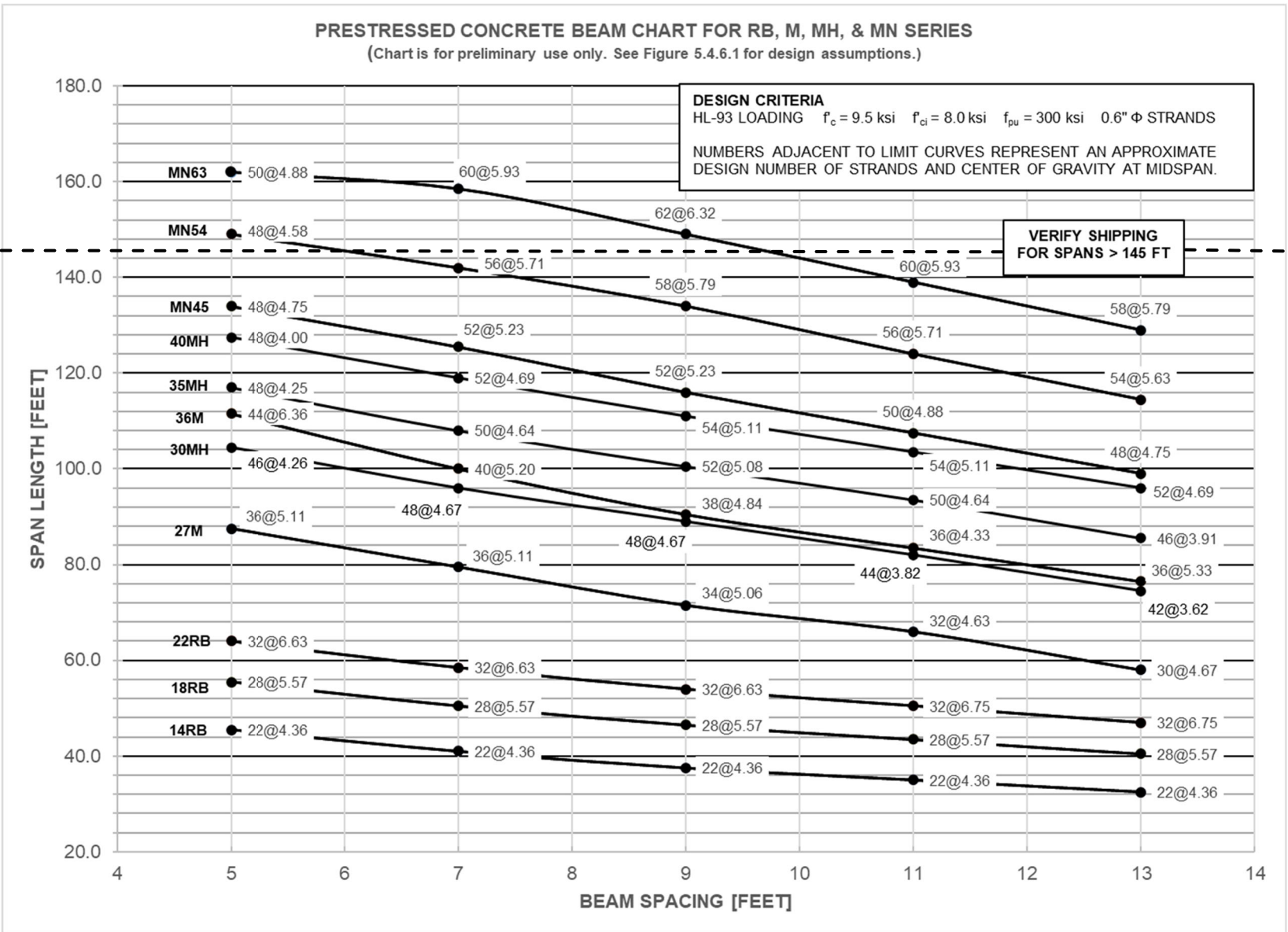
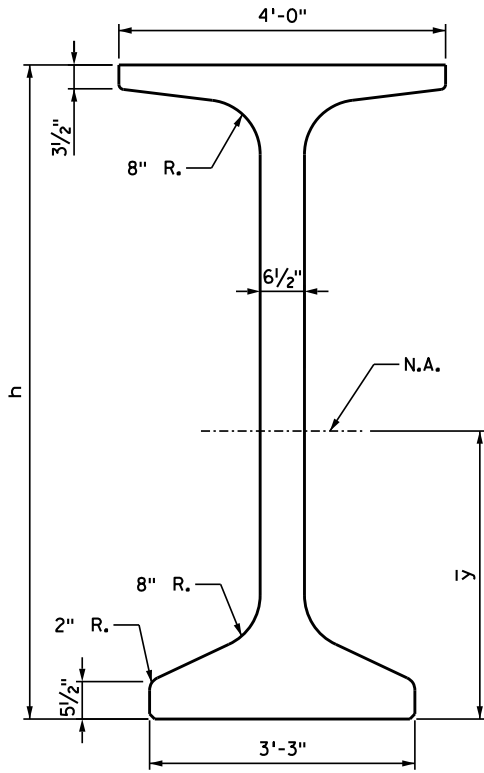


Figure 5.4.6.2 Beam Spg vs. Max Span Length Chart for RB, M, MH, and MN Series Beams



“MW” SERIES I-BEAM

DESIGN ASSUMPTIONS FOR PRETENSIONED CONCRETE BEAM CHART:  
2020 AASHTO LRFD Bridge Design Specifications, 9<sup>th</sup> Edition.

HL-93 Live Load

Beam Concrete:  $f'_c = 9.5 \text{ ksi}$   $f'_{ci} = 8.0 \text{ ksi}$   $w_{bm} = 0.155 \text{ kips/ft}^3$   
 $E_c = 1265 \sqrt{f'_c} + 1000 \text{ ksi}$

Deck Concrete:  $f'_c = 4.0 \text{ ksi}$   $E_c = 3987 \text{ ksi}$   
 $w_c = 0.145 \text{ kcf for } E_c \text{ computation}$   
 $w_c = 0.150 \text{ kcf for dead load computation}$

0.6" diameter low relaxation strands,  $E_s = 28,500 \text{ ksi}$   
 $f_{pu} = 300 \text{ ksi with initial pull of } 0.72f_{pu}$

Simple supports with six beams and deck without wearing course.  
Deck carries two Type S barriers with no sidewalk or median.  
Skew = 0 degrees.

Deck thickness for dead load calculations based on BDM Table 9.2.1.1.  
Effective deck thickness used for composite beam section properties is equal to total deck thickness minus 1/2" of wear.

2 1/2" average stool height used for dead load calculations.  
1 1/2" stool height used for composite beam section properties.

Barrier dead load is applied equally to all beams.  
Dead load includes 0.020 ksf future wearing course.

Approximate long term losses are used per LRFD 5.9.3.3.

Service Concrete Tensile Stress Limits:  
After Initial Losses:  $0.0948\sqrt{f'_{ci}} \leq 0.2 \text{ ksi}$   
After All Losses:  $0.19\sqrt{f'_c}$

Beam Properties

BEAM TYPE	h (in)	AREA (in <sup>2</sup> )	W ① (lb/ft)	$\bar{y}$ (in)	I (in <sup>4</sup> )	S <sub>b</sub> (in <sup>3</sup> )	A <sub>ct</sub> ② (in <sup>2</sup> )
82MW	82	1062	1143	38.37	1,010,870	26,345	609
96MW	96	1153	1241	45.02	1,486,510	33,019	655

① Based on 155 pounds per cubic foot.  
② Based on a 9" slab with 1/2" of wear and 1 1/2" stool.  
See AASHTO LRFD Art. 5.7.3.4.2 for A<sub>ct</sub> definition.

**Figure 5.4.6.3**  
**Precast Prestressed Concrete Beam Data for MW Series**

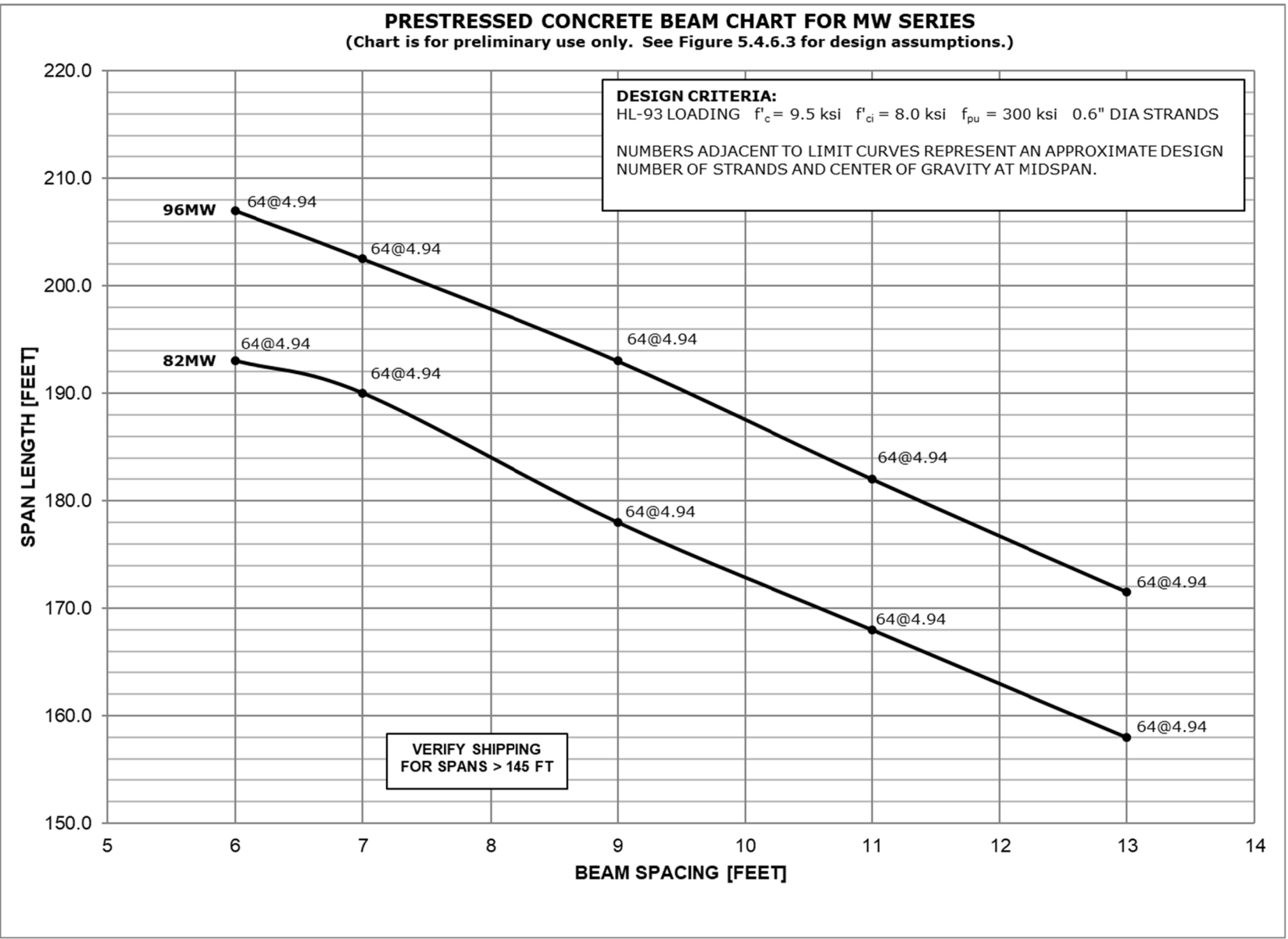


Figure 5.4.6.4  
Beam Spg vs. Max Span Length Chart for MW Series Beams

**5.4.8 Double-Tee Beams**

Pretensioned double-tees are not used anymore on Minnesota bridges. The standard Bridge Details Part II Figures 5-397.525 and 5-397.526 were archived in 2009.

**5.4.9 Inverted Tee Beams**

[Future manual content]

**5.5 Post-Tensioned Concrete**

Post-tensioned (PT) concrete structures have their prestressing steel stressed after the concrete has been placed and partially cured.

Post-tensioned concrete bridges are specialty structures. Poor detailing and poor construction practices can greatly reduce the service life of these structures. Designers should follow current practices recommended by the American Segmental Bridge Institute (ASBI) and the Post-Tensioning Institute (PTI).

Design segmental box girders and post-tensioned concrete slab bridges for zero tension under service loads.

For bridges where live load distribution has been determined using the approximate methods of LRFD Article 4.6.2.2 or 4.6.2.3, check tension stress after all losses using the Service III Limit State. For bridges where live load distribution has been determined using a refined analysis, check tension stress after all losses using the Service I Limit State.

**[5.9.3.2]**

Use gross section properties for design (i.e - do not transform prestressing strands to get transformed properties). Calculate the instantaneous losses (friction, anchor set, and elastic shortening losses) in accordance with LRFD Article 5.9.3.2. Use the refined method given in LRFD Article 5.9.3.4 to compute time-dependent prestress losses. Do not include any elastic gains caused by application of loads.

**[5.9.3.4]****5.5.1 PT Slab Bridges**

Post-tensioned concrete slab bridges are used for projects requiring spans longer than those efficiently accommodated with conventionally reinforced concrete slabs. The drawback to post-tensioned slabs is that they are more complex to design and construct. Elastic shortening and secondary bending moments due to post-tensioning are important design parameters for post-tensioned slab bridges. During construction a number of additional components are involved. They include the installation of post-tensioning ducts and anchorages, the pushing or pulling of strands through the ducts, the jacking of tendons, and grouting operations.

**5.5.2*****PT I-Girders***

Post-tensioned spliced I-girder bridges are not commonly used in Minnesota, but the MW series beams were developed with consideration of future use for spliced girder bridges. MnDOT will develop appropriate details as potential projects are identified.

**5.5.3 *PT Precast or  
Cast-In-Place Box  
Girders***

The depth of box girders should preferably be a minimum of  $1/18$  of the maximum span length.

Place vertical webs of box girders monolithic with the bottom slab.

**5.6 *Concrete  
Finishes and  
Coatings***

The finish or coating to be used on concrete elements will usually be determined when the Preliminary Bridge Plan is assembled. In general, provide a finish or coating consistent with the guidance given in the *Aesthetic Guidelines for Bridge Design Manual*.

A wide variety of surface finishes for concrete are used on bridge projects. These range from plain concrete to rubbed concrete to painted surfaces to form liners and stains. Plain concrete and rubbed concrete finishes are described in the MnDOT Spec. 2401. Painted and architectural surfaces must be described in the special provisions.

Specify graffiti protection for concrete elements with a coating system that has more than one color.

**5.7 Design  
Examples**

Section 5 concludes with design examples.

The examples are for the following:

- Article 5.7.1: Three-span continuous constant depth reinforced concrete slab superstructure.
- Article 5.7.2: Two-span pretensioned draped concrete I-beam superstructure.
- Article 5.7.3: Two-span pretensioned debonded concrete I-beam superstructure.
- Article 5.7.4: Three-span continuous haunched post-tensioned concrete slab superstructure. (Future content)

**5.7.1 Three-Span  
Continuous  
Constant Depth  
Reinforced  
Concrete Slab**

This example illustrates the design of a constant depth reinforced concrete slab bridge. The three continuous spans are 36'-0", 45'-0", and 36'-0" in length. The roadway width is 44'-0" with MnDOT Type S barrier railings for a total out-to-out width of 47'-0". The bridge is skewed 10 degrees and is supported by integral abutments at the ends. A plan view and typical sections of the bridge are shown in Figures 5.7.1.1 and 5.7.1.2.

After determining live load distribution factors, dead and live loads are computed at span tenth points. Next the live load deflection and the shear capacity of the section is checked. Then using Strength I, Service I, and Fatigue I design moments, the flexural reinforcement is sized. This is accomplished by:

- Providing adequate steel for strength
- Verifying that crack control checks are satisfied
- Checking fatigue stresses in the reinforcement
- Verifying that minimum reinforcement checks are satisfied

Finally, distribution and shrinkage and temperature reinforcement is sized.

Material and design parameters used in this example are:

Concrete Strength at 28 Days,  $f_c = 4.0$  ksi

Concrete Unit Weight,  $w_c = 0.150$  kcf (dead loads)

$w_c = 0.145$  kcf (modulus)

**[5.4.2.8]**

**[5.4.2.4]**

Concrete Density Modification Factor,  $\lambda = 1.0$

Concrete Aggregate Source Correction Factor,  $K_1 = 1.0$

Reinforcing Bars:

Yield Strength,  $f_y = 60$  ksi

Modulus of Elasticity,  $E_s = 29,000$  ksi

Top longitudinal bar concrete cover,  $d_{covtop} = 3.0$  in

Bottom longitudinal bar concrete cover,  $d_{covbot} = 1.5$  in

Weight of Future Wearing Surface = 0.020 ksf

Weight of concrete rail = 0.496 kip/ft

Assumed loss of slab thickness due to wear,  $h_{wear} = 0.5$  in

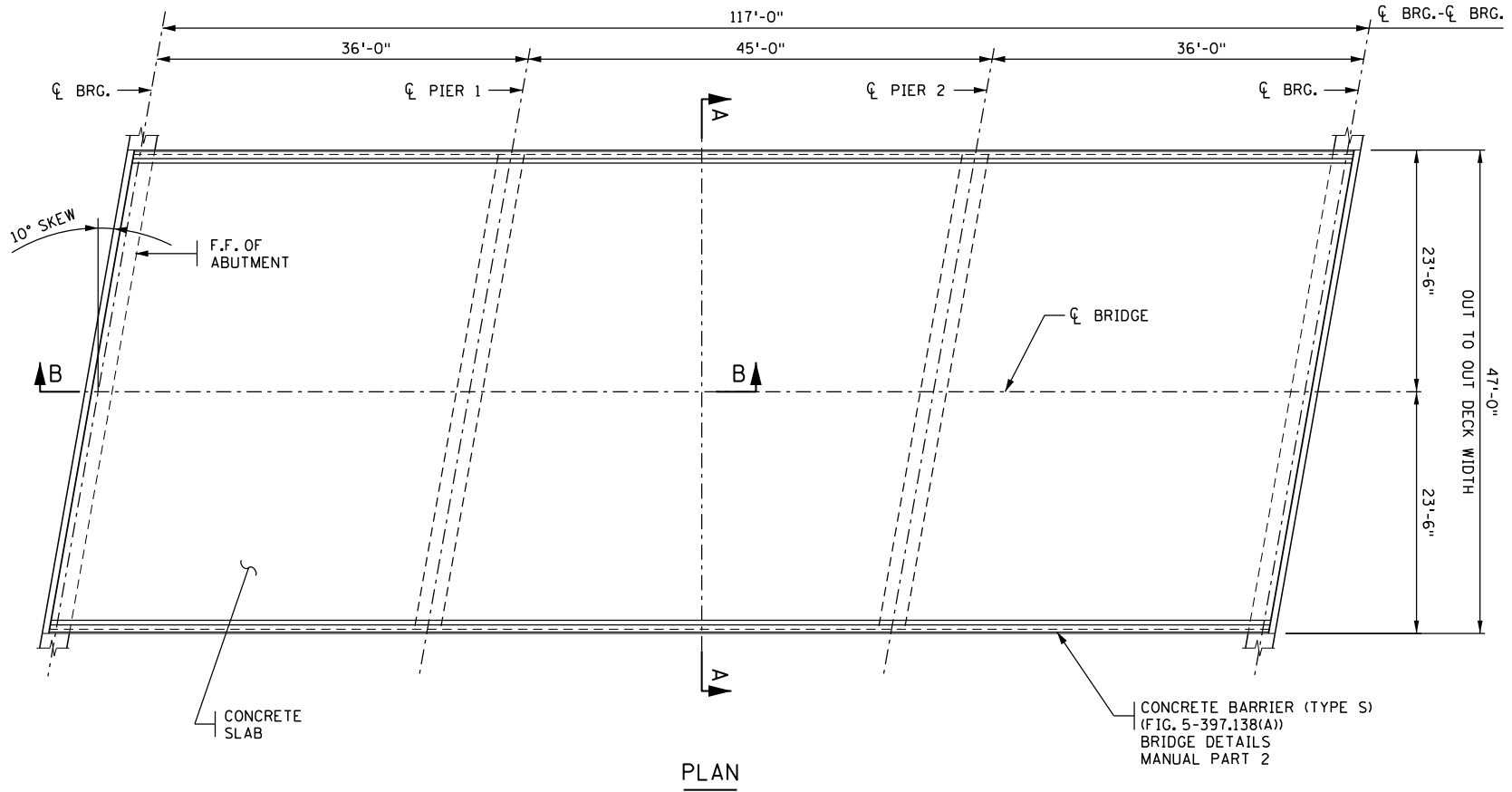
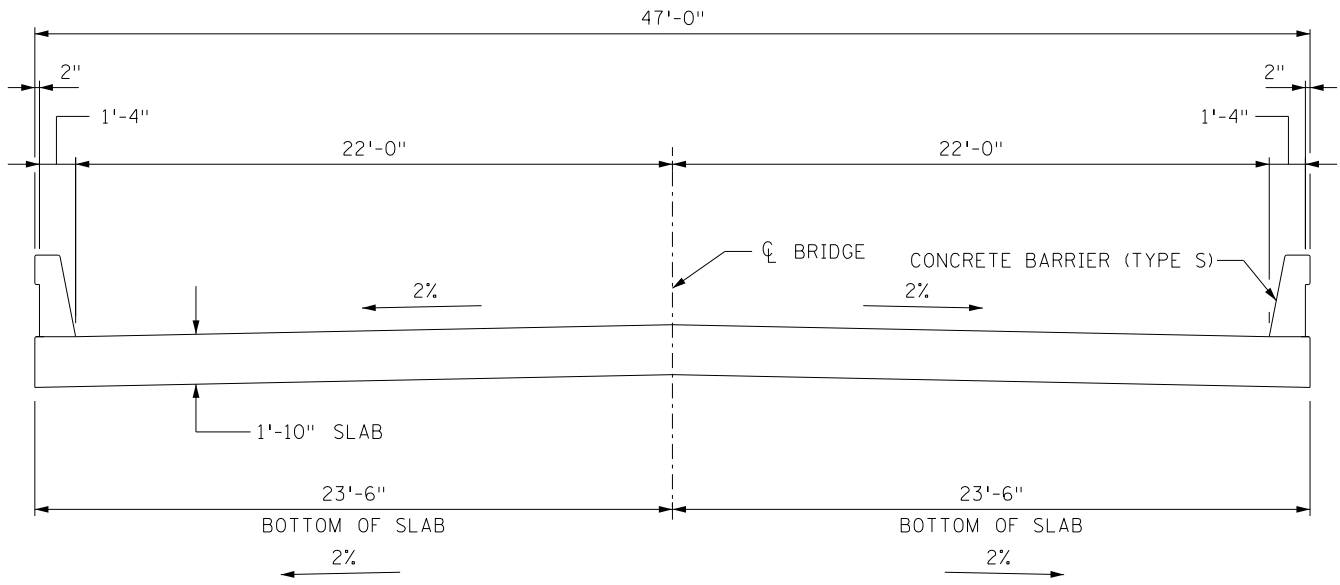
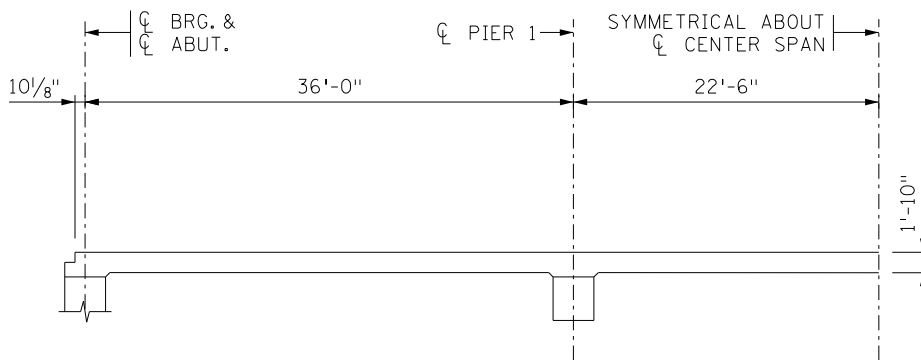


Figure 5.7.1.1





SECTION A-A



SECTION B-B

Figure 5.7.1.2

**A. Determine Slab Depth**

The minimum slab depth at midspan ( $h_{\min}$ ) is determined with the length of the longest span (S):

**[Table 2.5.2.6.3-1]**

$$h_{\min} = \frac{(S + 10)}{30} = \frac{(45 + 10)}{30} = 1.83 \text{ ft}$$

Choose slab depth  $h = 1.83 \text{ ft} = 22 \text{ in}$

**B. Determine Interior Strip Live Load Distribution Factors [4.6.2.3]**

The LRFD Specifications contain equations to determine the strip width that carries a lane of live load. Slab designs are performed on a strip one foot wide. The strip widths found with the LRFD equations are inverted to arrive at the live load distribution factor for a 1 foot wide strip (LLDF).

For interior strips multiple equations are evaluated to determine whether one or multiple live load lanes govern.

**Flexure – One Lane Loaded**

Multiple Presence Factors have been incorporated into the LRFD equations per LRFD C3.6.1.1.2.

**[4.6.2.3]**

Equivalent strip width (in),  $E = 10.0 + 5.0 \cdot \sqrt{L_1 \cdot W_1}$

Where:

$L_1$  = modified span length, equal to the lesser of actual span length or 60 ft.

$W_1$  = modified bridge width, equal to the lesser of actual bridge width or 30 ft for single lane loading.

For the 36 ft side spans:

$$E = 10.0 + 5.0 \cdot \sqrt{36 \cdot 30} = 174.3 \text{ in/lane}$$

$$\text{Therefore LLDF}_{\text{INT\_one}} = \frac{1}{\left(\frac{174.3}{12}\right)} = 0.069 \text{ lanes/ft} \quad \text{GOVERNS}$$

For the 45 ft center span:

$$E = 10.0 + 5.0 \cdot \sqrt{45 \cdot 30} = 193.7 \text{ in/lane}$$

$$\text{Therefore LLDF}_{\text{INT\_one}} = \frac{1}{\left(\frac{193.7}{12}\right)} = 0.062 \text{ lanes/ft}$$

**Flexure – Multiple Lanes Loaded****[4.6.2.3]**

$$\text{Equivalent strip width (in), } E = 84.0 + 1.44 \cdot \sqrt{L_1 \cdot W_1} \leq \frac{12 \cdot W}{N_L}$$

$$L_1 = 36 \text{ ft or } 45 \text{ ft}$$

$W_1$  = modified bridge width, equal to the lesser of actual bridge width or 60 ft for multilane loading.

$$W_1 = 47.00 \text{ ft}$$

$W$  is the actual bridge width = 47.00 ft

**[3.6.1.1.1]**

$$N_L = \frac{44}{12} = 3.7 \quad \text{Use } N_L = 3 \text{ lanes}$$

The upper limit on the equivalent strip width is:

$$\frac{12.0 \cdot W}{N_L} = \frac{12.0 \cdot 47.00}{3} = 188.0 \text{ in/lane}$$

For the 36 ft side spans:

$$E = 84.0 + 1.44 \cdot \sqrt{36 \cdot 47} = 143.2 \leq 188.0 \text{ in/lane}$$

$$\text{Therefore } LLDF_{INT\_mult} = \frac{1}{\left(\frac{143.2}{12}\right)} = 0.084 \text{ lanes/ft} \quad \text{GOVERNS}$$

For the 45 ft center span:

$$E = 84.0 + 1.44 \cdot \sqrt{45 \cdot 47} = 150.2 \leq 188.0 \text{ in/lane}$$

$$\text{Therefore } LLDF_{INT\_mult} = \frac{1}{\left(\frac{150.2}{12}\right)} = 0.080 \text{ lanes/ft}$$

**Reduction for Skew****[4.6.2.3]**

$$r = 1.05 - 0.25 \cdot \tan \theta = 1.05 - 0.25 \cdot \tan (10^\circ) = 1.006 > 1.0$$

Then  $r = 1.0$  No Reduction

To simplify the process of arriving at design forces, use the maximum interior strip live load distribution factor calculated for flexure for all locations.

$$LLDF_{INT} = 0.084 \text{ lanes/ft}$$

**[3.6.1.1.2]****Flexure – Fatigue Loading**

Divide the one lane LLDF by 1.2 to remove the Multiple Presence Factor.

For the 36 ft side spans:

$$\text{LLDF}_{\text{FAT}} = \frac{0.069}{1.2} = 0.058 \text{ lanes/ft} \quad \text{GOVERNS}$$

For the 45 ft center span:

$$\text{LLDF}_{\text{FAT}} = \frac{0.062}{1.2} = 0.052 \text{ lanes/ft}$$

**Shear and Deflection****[5.12.2.1]**

Although not required by AASHTO, MnDOT requires that slab type bridges be checked for shear using the same live load distribution factor calculated for deflection.

**[2.5.2.6.2]**

All design lanes are loaded and the entire slab assumed to resist the loads.

$$N_L = 3 \quad m = 0.85$$

The live load distribution factor for shear and deflection is:

$$\text{LLDF}_{\text{s}\Delta} = \frac{(\# \text{ of lanes}) \cdot (\text{MPF})}{(\text{deck width})} = \frac{3 \cdot 0.85}{47.00} = 0.054 \text{ lanes/ft}$$

**C. Determine  
Exterior Strip Live  
Load Distribution  
Factors  
[4.6.2.1.4]**

The exterior strip is assumed to carry one wheel line and a tributary portion of lane load.

Check if the equivalent strip is less than the maximum width of 72 inches.

$$\begin{aligned} E &= (\text{Distance from edge to inside of barrier}) + 12 + \left( \frac{\text{smallest int. } E}{4} \right) \\ &= 18 + 12 + \frac{143.2}{4} = 65.8 < 72.0 \text{ in} \quad \text{Use } E = 65.8 \text{ in} \end{aligned}$$

Compute the distribution factor associated with one truck wheel line:

$$\begin{aligned} \text{LLDF}_{\text{EXTT}} &= \frac{1 \text{ wheel line} \cdot \text{MPF}}{(2 \text{ wheel lines/lane}) \cdot (E/12)} = \frac{1 \cdot 1.2}{(2) \cdot (65.8/12)} \\ &= 0.109 \text{ lanes/ft} \end{aligned}$$

Compute the distribution factor associated with lane load on a 65.8 inch wide exterior strip:

$$\text{LLDF}_{\text{EXTL}} = \frac{\left(\frac{\text{deck width loaded}}{10 \text{ ft load width}}\right) \cdot \text{MPF}}{\text{(exterior strip width)}} = \frac{\left(\frac{65.8/12 - 18/12}{10}\right) \cdot 1.2}{(65.8/12)}$$

$$= 0.087 \text{ lanes/ft}$$

For simplicity, use the larger value for the exterior strip live load distribution factor for both load types when assembling design forces.

$$\text{LLDF}_{\text{EXT}} = 0.109 \text{ lanes/ft}$$

**D. Resistance  
Factors and Load  
Modifiers**  
[5.5.4.2]  
[1.3.3-1.3.5]

The following resistance factors are used for this example:

$\phi = 0.90$  for flexure and tension (tension-controlled reinforced concrete section is assumed, must be confirmed)

$\phi = 0.90$  for shear and torsion

The following load modifiers will be used for this example:

		Strength	Service	Fatigue
Ductility	$\eta_D$	1.0	1.0	1.0
Redundancy	$\eta_R$	1.0	1.0	1.0
Importance	$\eta_I$	1.0	n/a	n/a
	$\eta = \eta_D \cdot \eta_R \cdot \eta_I$	1.0	1.0	1.0

**E. Select  
Applicable Load  
Combinations and  
Load Factors**  
[3.4.1]

Three load combinations are applicable to the design of reinforced concrete slab superstructures:

STRENGTH I - Used to design for flexure and shear.

$$U = 1.0 \cdot [1.25 \cdot (\text{DC}) + 1.75 \cdot (\text{LL} + \text{IM})]$$

SERVICE I - Used for deflection and crack control checks.

$$U = 1.0 \cdot (\text{DC}) + 1.0 \cdot (\text{LL} + \text{IM})$$

FATIGUE I - Used to evaluate the reinforcing steel.

$$U = 1.0 \cdot 1.75 \cdot (\text{LL} + \text{IM})$$

EXTREME EVENT II - Used to check slab transverse reinforcement for a vehicle collision with the barrier.

$$U = 1.0 \cdot (\text{DC}) + 1.0 \cdot (\text{CT})$$

**F. Calculate Live Load Force Effects**  
**[3.6.1]**

The LRFD Specifications contain several live load components that are combined and scaled to generate the HL-93 design live loads. The components include: the design truck, design lane loading, design tandem, a pair of trucks, and a fatigue truck with fixed axle spacings.

For this example the following combinations will be investigated:

Design Truck w/Dynamic Load Allowance + Design Lane

Design Tandem w/Dynamic Load Allowance + Design Lane

Negative moment regions (modified per BDM Article 3.4.1):

$$1.25 \cdot [(\text{HL-93 Double Truck w/Dyn. Load Allow.}) + \text{Design Lane}]$$

Fatigue Truck

**[3.6.2.1]**

The dynamic load allowance (IM) has the following values:

IM = 15% when evaluating fatigue.

IM = 33% when evaluating all other limit states.

It is not applied to the lane live load.

**G. Calculate Dead Load Force Effects**

The dead load from the barriers is conservatively assumed to be fully carried by both interior and exterior strips.

**Interior Strip (1'-0" Wide)**

Slab:

$$W_{\text{slab}} = (1.0 \cdot 0.150 \cdot 1.83) = 0.275 \text{ kips/ft}$$

Barrier:

$$W_{\text{barrier}} = \frac{2 \cdot 0.496}{47.00} = 0.021 \text{ kips/ft}$$

Future Wearing Course:

Per BDM Article 3.3, include the future wearing course with DC loads.

$$W_{\text{fwc}} = \left( \frac{44}{47.00} \right) \cdot 0.020 = 0.019 \text{ kips/ft}$$

**[BDM 3.3]**

$$\text{Then } W_{\text{DC\_INT}} = 0.275 + 0.021 + 0.019 = 0.315 \text{ kips/ft}$$

**Exterior Strip (1'-0" Wide)**

Slab and future wearing course loads are the same as those for the interior strip.

Barrier:

$$W_{\text{barrier}} = \frac{0.496}{\left( \frac{65.8}{12} \right)} = 0.090 \text{ kips/ft}$$

$$\text{Then } W_{\text{DC\_EXT}} = 0.275 + 0.090 + 0.019 = 0.384 \text{ kips/ft}$$

### **H. Summary of Analysis Results**

A computer analysis was performed with a three-span continuous beam model to determine moments, shears, and deflections. The model included a pinned support at one of the abutments. A roller support was used at the pier supports and the other abutment.

Bending moment summaries obtained at different span locations are presented in Tables 5.7.1.1 through 5.7.1.4.

Shear information is presented in Tables 5.7.1.5 through 5.7.1.7.

Deflections are presented in Table 5.7.1.8. Dead load deflections are based on applying the interior strip slab and barrier loads (future wearing course is not included). The gross, uncracked section properties are used in the model to determine dead load deflections:

Slab depth  $h_{DCA} = 22$  in

Strip width  $W_{strip} = 12$  in

$$I_{DCA} = I_{gross} = \frac{W_{strip} \cdot (h_{DCA})^3}{12} = \frac{12 \cdot (22)^3}{12} = 10648 \text{ in}^4$$

Live load deflections are based on applying the HL-93 truck and lane load to a cracked section with an effective moment of inertia,  $I_{eff}$ , determined per BDM Article 5.3.5:

Assumed loss of slab thickness due to wear,  $h_{wear} = 0.5$  in

Slab depth  $h_{LLA} = 22 - 0.5 = 21.5$  in

$$I_{net} = \frac{W_{strip} \cdot (h_{LLA})^3}{12} = \frac{12 \cdot (21.5)^3}{12} = 9938 \text{ in}^4$$

#### **[BDM 5.3.5]**

$$I_{eff} = 0.5 \cdot I_{net} = 0.5 \cdot 9938 = 4969 \text{ in}^4$$

Moments, shears, and deflections are reported at span tenth points using the following nomenclature:

The number before the decimal point represents the span number and the number after the decimal point represents the tenth point within that span. For example,

Span Point 1.0 = 0 tenths point of Span 1 (centerline of bearing at abutment)

Span Point 2.5 = 5 tenths point of Span 2

Moments, shears, and deflections that appear later in the example are identified with bold numbers in the following tables.

**Table 5.7.1.1**  
**Moment Summary for One Lane of Live Load**

Span Point	Lane (kip-ft)	Truck (kip-ft)	Tandem (kip-ft)	Double Truck (kip-ft)	+ Fatigue (kip-ft)	- Fatigue (kip-ft)
1.0	0	0	0	-	0	0
1.1	34	152	145	-	113	-15
1.2	59	248	248	-	192	-29
1.3	76	297	308	-	239	-44
1.4	85	305	330	-	<b>255</b>	<b>-58</b>
1.5	86	294	324	-	246	-73
1.6	78/-42	278/-137	294/-116	-130	214	-88
1.7	62/-49	226/-159	236/-136	-152	173	102
1.8	-56	-182	-155	-174	120	-117
1.9	-75	-205	-175	-196	61	-164
2.0	-119	-252	-194	-232	<b>33</b>	<b>-252</b>
2.1	-62	-154	-143	-154	71	-142
2.2	35/-37	164/-132	183/-122	-132	140	-90
2.3	66/-36	259/-109	267/-101	-126	197	-74
2.4	86	314	318	-	249	-59
2.5	92	323	332	-	259	-44

**Table 5.7.1.2**  
**Moment Summary – Interior Strip (per foot width)**

Span Point	M <sub>DC</sub> (kip-ft)	* Truck + Lane (kip-ft)	* Tandem + Lane (kip-ft)	* 1.25 (Double Truck + Lane) (kip-ft)
1.0	0.0	0	0	-
1.1	12.9	20	19	-
1.2	21.8	33	33	-
1.3	26.8	40	41	-
1.4	<b>27.8</b>	41	44	-
1.5	24.8	40	43	-
1.6	17.3	38/-19	39/-16	-23
1.7	5.8	30/-22	32/-19	-26
1.8	-9.6	-25	-22	-30
1.9	-29.0	-29	-26	-35
2.0	<b>-52.4</b>	-38	-32	-45
2.1	-24.1	-22	-21	-28
2.2	-1.9	21/-18	23/-17	-22
2.3	14.0	34/-15	35/-14	-21
2.4	23.8	42	43	-
2.5	27.3	44	45	-

\* Includes Dynamic Load Allowance (IM) and Live Load Distribution Factor.



**Table 5.7.1.3**  
**Moment Summary – Exterior Strip (per foot width)**

Span Point	M <sub>DC</sub> (kip-ft)	* Truck + Lane (kip-ft)	* Tandem + Lane (kip-ft)	* 1.25 (Double Truck + Lane) (kip-ft)
1.0	0.0	0	0	-
1.1	15.7	26	25	-
1.2	26.6	42	42	-
1.3	32.7	51	53	-
1.4	33.9	53	57	-
1.5	30.3	52	56	-
1.6	21.1	49/-24	51/-21	-29
1.7	7.1	40/-28	41/-25	-34
1.8	-11.7	-32	-29	-39
1.9	-35.4	-38	-34	-46
2.0	-63.9	-50	-41	-58
2.1	-29.3	-29	-27	-36
2.2	-2.3	28/-23	30/-22	-29
2.3	17.1	45/-20	46/-19	-28
2.4	29.0	55	55	-
2.5	33.3	57	58	-

\* Includes Dynamic Load Allowance (IM) and Live Load Distribution Factor.

**Table 5.7.1.4**  
**Moment Load Combinations**

Span Point	Service I		Strength I	
	Interior (kip-ft)/ft	Exterior (kip-ft)/ft	Interior (kip-ft)/ft	Exterior (kip-ft)/ft
1.0	0	0	0	0
1.1	33	42	51	65
1.2	55	69	85	107
1.3	68	86	105	134
1.4	<b>72</b>	<b>91</b>	<b>112</b>	<b>142</b>
1.5	68	86	106	136
1.6	56/-6	72/-8	90/-19	116/-24
1.7	<b>38/-20</b>	<b>48/-27</b>	<b>63/-38</b>	<b>81/-51</b>
1.8	<b>11/-40</b>	<b>15/-51</b>	<b>25/-65</b>	<b>33/-83</b>
1.9	<b>-64</b>	<b>-81</b>	<b>-98</b>	<b>-125</b>
2.0	<b>-97</b>	<b>-122</b>	<b>-144</b>	<b>-181</b>
2.1	<b>-52</b>	<b>-65</b>	<b>-79</b>	<b>-100</b>
2.2	<b>21/-24</b>	<b>28/-31</b>	<b>38/-41</b>	<b>50/-54</b>
2.3	<b>49/-7</b>	<b>63/-11</b>	<b>79/-19</b>	<b>102/-28</b>
2.4	67	84	105	132
2.5	<b>72</b>	<b>91</b>	<b>113</b>	<b>143</b>

**Table 5.7.1.5**  
**Shear Summary – One Lane**

Span Point	Lane (kips)	Truck (kips)	Tandem (kips)
1.0	10.5	50.2	46.6
1.1	8.4	42.1	40.4
1.2	6.5	34.4	34.4
1.3	4.9	27.5	28.5
1.4	4.2	21.1	22.9
1.5	5.4	23.3	26.5
1.6	6.9	30.5	31.8
1.7	8.6	37.6	36.7
1.8	10.5	44.2	41.1
1.9	12.6	50.6	45.0
2.0	15.8	57.9	48.4
2.1	13.0	51.0	44.4
2.2	10.5	43.5	39.5
2.3	8.2	35.7	34.1
2.4	6.3	28.0	28.3
2.5	4.7	20.9	22.4

**Table 5.7.1.6**  
**Shear Summary (per foot width)**

Span Point	* $V_{DC}$ (kips)	** Truck + Lane (kips)	** Tandem + Lane (kips)
1.0	5.1	4.2	3.9
1.1	3.8	3.5	3.4
1.2	2.4	2.8	2.8
1.3	1.0	2.2	2.3
1.4	0.4	1.7	1.8
1.5	1.8	2.0	2.2
1.6	3.2	2.6	2.7
1.7	4.5	3.2	3.1
1.8	5.9	3.7	3.5
1.9	7.3	4.3	3.9
2.0	8.7	5.0	4.3
2.1	6.9	4.4	3.9
2.2	5.2	3.7	3.4
2.3	3.5	3.0	2.9
2.4	1.7	2.4	2.4
2.5	0.0	1.8	1.9

\* Conservatively based on exterior strip dead load shear values.

\*\* Includes Dynamic Load Allowance (IM) and 0.054 Distribution Factor.

**Table 5.7.1.7**  
**Shear Summary – Load Combinations**

Span Point	SERVICE I (kips)	STRENGTH I (kips)
1.0	9.3	<b>13.7</b>
1.1	7.3	10.9
1.2	5.2	7.9
1.3	3.3	5.3
1.4	2.2	3.7
1.5	4.0	6.1
1.6	5.9	8.7
1.7	7.7	11.2
1.8	9.6	13.9
1.9	11.6	16.7
2.0	13.7	<b>19.6</b>
2.1	11.3	16.3
2.2	8.9	13.0
2.3	6.5	9.6
2.4	4.1	6.3
2.5	1.9	3.3

**Table 5.7.1.8**  
**Deflection Summary**

Span Point	* Dead Load Deflection (in)	** Lane LL Deflection (in)	** Truck LL Deflection (in)
1.0	0.000	0.000	0.000
1.1	0.040	0.017	0.075
1.2	0.073	0.032	0.140
1.3	0.096	0.043	0.190
1.4	<b>0.105</b>	0.050	0.218
1.5	0.101	<b>0.052</b>	<b>0.223</b>
1.6	0.085	0.048	0.206
1.7	0.060	0.040	0.170
1.8	0.032	0.028	0.117
1.9	0.009	0.014	0.057
2.0	0.000	0.000	0.000
2.1	0.019	0.020	0.087
2.2	0.057	0.042	0.181
2.3	0.097	0.060	0.262
2.4	0.125	0.072	0.314
2.5	<b>0.136</b>	<b>0.076</b>	<b>0.331</b>

\* Based on  $I_{gross}$ . Includes selfweight and barrier loading for an interior strip.

\*\* Based on  $I_{effective} = 0.5 \cdot I_{net}$ . Includes Dynamic Load Allowance (IM) for truck deflection only and Live Load Distribution Factor.

**I. Live Load  
Deflection  
[2.5.2.6]**

Live load deflection is checked before the flexural design to confirm that the 22 in slab depth chosen earlier is adequate. To prevent serviceability problems, MnDOT uses the AASHTO optional maximum live load deflection limit:

$$\Delta_{LLmax} = \frac{\text{Span}}{800}$$

$$\text{For Spans 1 and 3, } \Delta_{LLmax13} = \frac{36 \cdot 12}{800} = 0.540 \text{ in}$$

$$\text{For Span 2, } \Delta_{LLmax2} = \frac{45 \cdot 12}{800} = 0.675 \text{ in}$$

**[3.6.1.3.2]**

Using the Table 5.7.1.8 live load deflection values, check the actual maximum live load deflections for the design truck alone and the design lane load plus 25% of truck load:

Midspans 1 and 3

$$\text{Truck: } \Delta_{LLtr13} = 0.223 \text{ in} < 0.540 \text{ in}$$

$$\begin{aligned} \text{Lane + 25\% Truck: } \Delta_{LLlanetr13} &= 0.052 + 0.25 (0.223) \\ &= 0.108 \text{ in} < 0.540 \text{ in} \end{aligned}$$

Midspan 2

$$\text{Truck: } \Delta_{LLtr2} = 0.331 \text{ in} < 0.675 \text{ in}$$

$$\begin{aligned} \text{Lane + 25\% Truck: } \Delta_{LLlanetr2} &= 0.076 + 0.25 (0.331) \\ &= 0.159 \text{ in} < 0.540 \text{ in} \end{aligned}$$

**J. Shear in Slab  
[5.12.8.6]  
[BDM 5.3.2]  
[5.7.3.2]**

Check the one-way shear capacity of the slab. This check is done before the flexural design to confirm that the 22 in slab depth chosen earlier is adequate.

**Critical Section**

Shear should be checked at all sections. In our case the governing location is at Pier 1 (Span Point 2.0). Although the critical section for shear is located  $d_v$  from the face of the pier, we will conservatively make the check at the pier centerline.

**[5.7.2.8]**

The effective shear depth  $d_v$  is the distance between the internal tension and compression force components to resist flexure, which is unknown at this point in the design.

The top bar clear cover,  $d_{covtop}$ , is 3.0 inches.

Assuming a #7 bar, the shear depth need not be less than

$$\begin{aligned} 0.9 \cdot d_e &= 0.9 \cdot [h - d_{\text{covtop}} - 0.5 \cdot d_{\text{bar}}] \\ &= 0.9 \cdot [22 - 3 - 0.5 \cdot 0.875] = 16.71 \text{ in} \end{aligned}$$

or

$$0.72 \cdot h = 0.72 \cdot (22) = 15.84 \text{ in}$$

Use  $d_v = 16.71$  in

The shear load at Span Point 2.0 from Table 5.7.1.7 is:

$$V_U = 19.6 \text{ kips}$$

### [5.7.3.3]

#### Nominal Shear Resistance

The nominal shear resistance is the sum of the contributions from the concrete and steel.

$$V_n = V_c + V_s$$

It can be no more than:

$$V_n \leq 0.25 \cdot f'_c \cdot b_v \cdot d_v = 0.25 \cdot 4.0 \cdot 12 \cdot 16.71 = 200.5 \text{ kips}$$

For calculation of the concrete contribution, use  $\beta = 2.0$  per BDM Article 5.3.2. If shear reinforcement is found necessary, thicken the slab to eliminate the need for shear reinforcement:

$$\begin{aligned} V_c &= 0.0316 \cdot \beta \cdot \lambda \cdot \sqrt{f'_c} \cdot b_v \cdot d_v \\ &= 0.0316 \cdot 2.0 \cdot 1.0 \cdot \sqrt{4.0} \cdot 12 \cdot 16.71 = 25.3 \text{ kips} \end{aligned}$$

Without shear reinforcement,  $V_s = 0$

The nominal shear capacity of the slab is:

$$V_n = 25.3 + 0 = 25.3 \text{ kips} < 200.5 \text{ kips} \quad \text{OK}$$

Check if the shear resistance is greater than the shear demand:

### [5.5.4.2]

For shear in reinforced concrete sections,  $\phi_v = 0.90$

$$V_r = \phi_v \cdot V_n = 0.90 \cdot (25.3) = 22.8 \text{ kips} > 19.6 \text{ kips} \quad \text{OK}$$

**K. Design Positive  
Moment  
Reinforcement**

[5.6.2.2]

[5.6.3.2]

Determine the required area of flexural reinforcement to satisfy the Strength I Load Combination.

**Flexural Resistance**

Assume a rectangular stress distribution and solve for the required area of reinforcing based on the factored moment,  $M_u$ , and an assumed depth,  $d_s$ . Also assume the section is tension-controlled and the flexural resistance factor  $\phi$  is 0.90.

For  $f'_c = 4.0$  ksi,  $\beta_1 = 0.85$ , and  $\alpha_1 = 0.85$

$$M_u = \phi \cdot M_n = \phi \cdot A_s \cdot f_y \cdot \left(d_s - \frac{a}{2}\right)$$

$$a = \beta_1 c = \frac{A_s \cdot f_y}{\alpha_1 \cdot f'_c \cdot b}$$

$$M_u = \phi \cdot A_s \cdot f_y \cdot \left(d_s - \frac{A_s \cdot f_y}{2 \cdot \alpha_1 \cdot f'_c \cdot b}\right)$$

$$M_u = 0.90 \cdot A_s \cdot (60) \cdot \left(d_s - \frac{A_s \cdot 60}{2 \cdot 0.85 \cdot 4 \cdot 12}\right) \cdot \left(\frac{1}{12}\right)$$

$$3.309 \cdot A_s^2 - 4.5 \cdot d_s \cdot A_s + M_u = 0$$

$$A_s = \frac{4.5 \cdot d_s - \sqrt{20.25 \cdot d_s^2 - 13.236 \cdot M_u}}{6.618}$$

For the calculation of  $d_s$ , assume #7 bars for the interior strip and #8 bars for the exterior strip.

$$d_{s\_int} = 22 - 0.50 - 1.5 - 0.5 \cdot (0.875) = 19.56 \text{ in}$$

$$d_{s\_ext} = 22 - 0.50 - 1.5 - 0.5 \cdot (1.0) = 19.50 \text{ in}$$

Trial reinforcement information for Span Points 1.4 and 2.5 is provided in Table 5.7.1.9. After evaluating the areas of steel required, a layout based on a bar spacing of 5 inches was selected.

As the positive moment reduces towards the supports, the required area of steel is reduced, so theoretically the bar area or spacing could change multiple times along the length of the bridge. Later in this example, using the guidance in BDM Article 5.3.4, we will design for one change between the point of maximum positive moment and adjacent supports in each span. Half of the bottom longitudinal bars will be dropped, resulting in a new bar spacing of 10 inches.

**Table 5.7.1.9**  
**Trial Bottom Longitudinal Reinforcement**

Span Point	Interior Strip					Exterior Strip				
	$M_u$ (k-ft)/ft	$d_s$ (in)	Reqd. $A_s$ (in <sup>2</sup> /ft)	Trial Bars	Provided $A_s$ (in <sup>2</sup> /ft)	$M_u$ (k-ft)/ft	$d_s$ (in)	Reqd. $A_s$ (in <sup>2</sup> /ft)	Trial Bars	Provided $A_s$ (in <sup>2</sup> /ft)
1.4	112	19.56	1.34	#7 @ 5	1.44	142	19.50	1.73	#8 @ 5	1.90
2.5	113	19.56	1.35	#7 @ 5	1.44	143	19.50	1.74	#8 @ 5	1.90

**[5.5.4.2]**

Validate the assumption of 0.9 for resistance factor for the interior strip:

Calculate the depth of the Whitney stress block.

$$a = \frac{A_s \cdot f_y}{\alpha_1 \cdot f'_c \cdot b} = \frac{1.44 \cdot 60}{0.85 \cdot 4 \cdot 12} = 2.12 \text{ in}$$

The depth of the section in compression is:

$$c = \frac{a}{\beta_1} = \frac{2.12}{0.85} = 2.49 \text{ in}$$

**[5.6.2.1]**

Concrete compression strain limit  $\epsilon_c = 0.003$

Reinforcement tension-controlled strain limit  $\epsilon_{tl} = 0.005$

**[C5.6.2.1]**

Referring to AASHTO LRFD Figure C5.6.2.1-1, determine the net tensile strain in the extreme tension steel at nominal resistance,  $\epsilon_t$ , using similar triangles:

$$\epsilon_t = (d_s - c) \cdot \left( \frac{\epsilon_c}{c} \right) = (19.56 - 2.49) \cdot \left( \frac{0.003}{2.49} \right) = 0.0206 > 0.005$$

Therefore, the interior strip section is tension-controlled and  $\phi = 0.90$ .

Similarly for the exterior strip:

$$a = 2.79 \text{ in}$$

$$c = 3.28 \text{ in}$$

$$\epsilon_t = 0.0148 > 0.005$$

Therefore, the exterior strip section is also tension-controlled and  $\phi = 0.90$ .

[5.6.7]

**Crack Control**

To ensure that cracking is limited to small cracks that are well distributed, a limit is placed on the spacing of the reinforcing steel. LRFD Equation 5.6.7-1 defines the maximum spacing permitted:

$$s \leq s_{max} = \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

At Span Points 1.4 and 2.5 of the interior strip, the Service I positive moment  $M_s$  is 72 kip-ft.

At Span Points 1.4 and 2.5 of the exterior strip, the Service 1 positive moment  $M_s$  is 91 kip-ft.

[5.4.2.4 & 5.6.1]

The stress in the reinforcement is found using a cracked section analysis with the trial reinforcement. To simplify the calculations, the section is assumed to be singly reinforced.

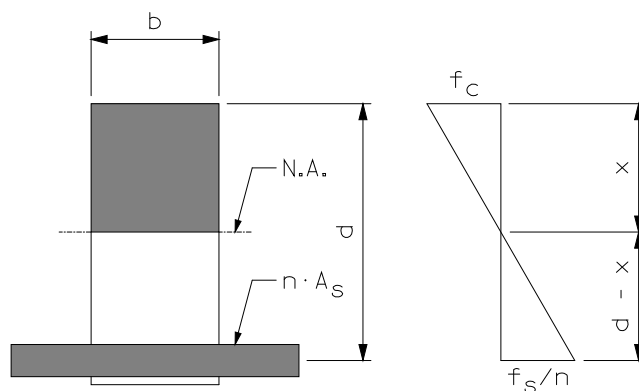
$$E_c = 120,000 \cdot K_1 \cdot (w_c)^2 \cdot (f'_c)^{0.33}$$

$$= 120,000 \cdot 1.0 \cdot (0.145)^2 \cdot (4.0)^{0.33} = 3987 \text{ ksi}$$

$$n = \frac{E_s}{E_c} = \frac{29,000}{3987} = 7.27$$

$$n \cdot A_s = 7.27 \cdot (1.44) = 10.47 \text{ in}^2$$

Referring to Figure 5.7.1.3, determine the location of the neutral axis for the interior strip:



**Figure 5.7.1.3**

$$b \cdot x \cdot \frac{x}{2} = n \cdot A_s \cdot (d_s - x)$$

$$\frac{(12) \cdot x^2}{2} = 7.27 \cdot 1.44 \cdot (19.56 - x)$$

Solving,  $x = 5.03 \text{ in}$



Determine the lever arm between service load flexural force components.

$$j \cdot d_s = d_s - \frac{x}{3} = 19.56 - \frac{5.03}{3} = 17.88 \text{ in}$$

The maximum allowable tension stress  $f_{ss\max}$  is:

$$f_{ss\max} = 0.60 \cdot f_y = 0.60 \cdot 60 = 36.0 \text{ ksi}$$

Compute the actual stress in the reinforcement.

$$f_{ss} = \frac{M_s}{A_s \cdot j \cdot d_s} = \frac{72 \cdot 12}{1.44 \cdot (17.88)} = 33.6 \text{ ksi} < 36.0 \text{ ksi} \quad \text{OK}$$

$$d_c = d_{\text{covbot}} + 0.5 \cdot d_b = 1.5 + 0.5 \cdot 0.875 = 1.94 \text{ in}$$

$$\beta_s = 1 + \frac{d_c}{0.7 \cdot (h - h_{\text{wear}} - d_c)} = 1 + \frac{1.94}{0.7 \cdot (22 - 0.50 - 1.94)} = 1.14$$

**[BDM 5.3.2]**

Per BDM Art. 5.3.2, assume Class 2 exposure condition. Use  $\gamma_e = 0.75$

$$s_{\max} = \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = \frac{700 \cdot 0.75}{1.14 \cdot 33.6} - 2 \cdot 1.94 = 9.8 \text{ in} > 5 \text{ in} \quad \text{OK}$$

Similarly, for the exterior strip:

$$x = 5.65 \text{ in}$$

$$j \cdot d_s = 17.62 \text{ in}$$

$$f_{ss} = 32.6 \text{ ksi} < 36.0 \text{ ksi} \quad \text{OK}$$

$$d_c = 2.00 \text{ in}$$

$$\beta_s = 1.15$$

$$s_{\max} = 10.0 \text{ in} > 5 \text{ in} \quad \text{OK}$$

**[5.5.3]**

**Fatigue**

The stress range in the reinforcement is computed using the Fatigue I load combination and compared against code limits to ensure adequate fatigue resistance is provided.

**[Table 3.4.1-1]**

Fatigue I:  $U = 1.0 \cdot 1.75$  (LL + IM)

**[3.6.2.1]**

The dynamic load allowance for fatigue,  $IM = 15\%$

The largest fatigue live load moment range is at Span Point 1.5. However, the magnitude of  $f_{\min}$  affects the constant-amplitude fatigue threshold value  $(\Delta F)_{\text{TH}}$ , so check fatigue at Span Points 1.4, 1.5, 2.4, and 2.5.

At Span Point 1.4 the one lane fatigue moments are:

Maximum positive moment = 255 kip-ft

Maximum negative moment = -58 kip-ft

Multiply the one lane moments by the dynamic load allowance and the fatigue live load distribution factor to get the fatigue moments for a 1 foot wide strip:

$$\text{Fatigue LL } M_{\text{fatmax}} = 255 \cdot 1.15 \cdot 0.058 = 17.0 \text{ kip-ft}$$

$$\text{Fatigue LL } M_{\text{fatmin}} = -58 \cdot 1.15 \cdot 0.058 = -3.9 \text{ kip-ft}$$

The unfactored dead load moment at Span Point 1.4 is 27.8 kip-ft.

### [5.5.3.1]

AASHTO requires the use of cracked section properties where the sum of the unfactored dead load stress and Fatigue I stress exceeds  $0.095\sqrt{f'_c}$  in tension. Otherwise uncracked section properties may be used. For simplicity, conservatively assume the section is cracked.

Then using the equation previously used to check crack control stresses, determine the fatigue live load stress range  $\Delta f$ :

$$\Delta f = \frac{M_{\text{fatmax}} - M_{\text{fatmin}}}{A_s \cdot j \cdot d_s} = \frac{[17.0 - (-3.9)] \cdot 12}{1.44 \cdot 17.88} = 9.7 \text{ ksi}$$

For the fatigue I load combination, the factored live load stress is:

$$\gamma \cdot \Delta f = 1.75 \cdot 9.7 = 17.0 \text{ ksi}$$

### [5.5.3.2]

Calculate the constant-amplitude fatigue threshold  $(\Delta F)_{\text{TH}}$ :

$$f_{\text{min}} = \frac{\gamma \cdot M_{\text{fatmin}} + M_{\text{DC}}}{A_s \cdot j \cdot d_s} = \frac{[1.75 \cdot (-3.9) + 27.8] \cdot 12}{1.44 \cdot 17.88} = 9.8 \text{ ksi}$$

$$(\Delta F)_{\text{TH}} = 26 - \frac{22 \cdot f_{\text{min}}}{f_y} = 26 - \frac{22 \cdot 9.8}{60} = 22.4 \text{ ksi} > 17.0 \text{ ksi} \quad \text{OK}$$

Span Points 1.5, 2.4, and 2.5 were checked in a similar way. A summary of the results is shown in Table 5.7.1.10.

**Table 5.7.1.10**  
**Fatigue Stress Check Summary**

Span Point	$M_{fatmax}$ (k-ft)	$M_{fatmin}$ (k-ft)	$\Delta f$ (ksi)	$\gamma \cdot \Delta f$ (ksi)	$f_{min}$ (ksi)	$(\Delta F)_{TH}$ (ksi)
1.4	17.0	-3.9	9.7	17.0	9.8	22.4
1.5	16.4	-4.9	9.9	17.3	7.6	23.2
2.4	16.6	-3.9	9.6	16.8	7.9	23.1
2.5	17.3	-2.9	9.4	16.5	10.4	22.2

**[5.6.3.3]**

**Check Minimum Reinforcement**

To prevent a brittle failure, adequate flexural reinforcement needs to be placed in the cross section. For this check, zero wear is conservatively assumed.

Check that the reinforcement can carry the smaller of:

- Cracking moment,  $M_{cr}$
- $1.33 \cdot M_u$

At Span Point 1.4:

$$1.33 \cdot M_u = 1.33 \cdot 112 = 149.0 \text{ kip-ft}$$

At Span Point 2.5:

$$1.33 \cdot M_u = 1.33 \cdot 113 = 150.3 \text{ kip-ft}$$

**[5.4.2.6]**

$$f_r = 0.24 \cdot \lambda \cdot \sqrt{f'_c} = 0.24 \cdot 1.0 \cdot \sqrt{4.0} = 0.48 \text{ ksi}$$

$$y_t = \frac{h}{2} = \frac{22.0}{2} = 11.0 \text{ in}$$

$$I_g = \frac{1}{12} \cdot b \cdot h^3 = \frac{1}{12} \cdot 12 \cdot (22)^3 = 10648 \text{ in}^4$$

$$S_c = \frac{I_g}{y_t} = \frac{10648}{11.0} = 968 \text{ in}^3$$

For non-precast segmental structures,  $\gamma_1 = 1.6$

MnDOT uses AASHTO M31 (ASTM A615) Grade 60 reinforcement in concrete bridge structures, so  $\gamma_3 = 0.67$

$$M_{cr} = \gamma_3 \cdot (\gamma_1 \cdot f_r \cdot S_c) = \frac{0.67 \cdot (1.6 \cdot 0.48 \cdot 968)}{12} = 41.5 \text{ kip-ft} \quad \text{GOVERNS}$$

$$M_r = \phi \cdot A_s \cdot f_y \cdot \left( d_s - \frac{a}{2} \right)$$

$$M_r = 0.9 \cdot (1.44) \cdot (60) \cdot \left( 19.56 - \frac{2.12}{2} \right) \cdot \frac{1}{12}$$

$$M_r = 119.9 \text{ kip-ft} > M_{cr} = 41.5 \text{ kip-ft}$$

OK

Use #7 bars at 5 inches at Span Points 1.4 and 2.5

Similarly for the exterior strip:

At Span Point 1.4,  $1.33 \cdot M_u = 188.9$  kip-ft

At Span Point 2.5,  $1.33 \cdot M_u = 190.2$  kip-ft

$M_{cr} = 41.5$  kip-ft

$M_r = 154.8$  kip-ft > 41.5 kip-ft

OK

Use #8 bars at 5 inches at Span Points 1.4 and 2.5

**[5.10.8.1.2a]**

**[5.10.8.1.2b]**

### Bar Cutoff Location

Determine the location where the 5 inch spacing can be increased to 10 inches. The moment capacity of #7 bars at 10 inches ( $A_s = 0.72$  in<sup>2</sup>) for positive flexure is:

$$M_r = \phi \cdot A_s \cdot f_y \cdot \left( d_s - \frac{a}{2} \right)$$

$$M_r = 0.9 \cdot (0.72) \cdot (60) \cdot \left( 19.56 - \frac{0.72 \cdot (60)}{2 \cdot (0.85) \cdot (4) \cdot (12)} \right) \cdot \frac{1}{12}$$

$$= 61.7 \text{ kip-ft}$$

For the interior strip, the positive bending moments are:

Span Point	$M_{\text{Strength I}}$ (kip-ft)/ft	$M_{\text{Service I}}$ (kip-ft)/ft
1.7	63	38
1.8	25	11
2.2	38	21
2.3	79	49

Knowing that span points are 3.6 feet apart in Span 1 and 4.5 feet apart in Span 2, the drop point locations which meet the positive Strength I bending moment of 61.7 kip-ft can be found.

For Span 1, interpolate between Span Points 1.7 and 1.8:

$$1.7 + \left( \frac{63 - 61.7}{63 - 25} \right) \cdot 0.1 = 1.70 \text{ or } 10.80 \text{ ft from Pier 1 centerline.}$$

For Span 2, interpolate between Span Points 2.2 and 2.3:

$$2.2 + \left( \frac{61.7 - 38}{79 - 38} \right) \cdot 0.1 = 2.26 \text{ or } 11.70 \text{ ft from Pier 1 centerline.}$$

The reinforcement must also meet the serviceability requirements at the theoretical drop point. Determine the drop point location based on the crack control requirements and compare with the drop points based on strength to see which ones govern.

**[5.6.7]**

For #7 bars @ 10", ( $A_s = 0.72 \text{ in}^2$ ), and  $d_c = 1.94 \text{ in}$ :

$$s = 10 \text{ in} \leq \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

where

$$f_{ss} = \frac{M_{\text{drop}}}{A_s \cdot j \cdot d_s}$$

Determine neutral axis:

$$n \cdot A_s = 7.27 \cdot 0.72 = 5.23 \text{ in}^2$$

$$b \cdot x \cdot \frac{x}{2} = n \cdot A_s \cdot (d_s - x)$$

$$\frac{12 \cdot x^2}{2} = 5.23 \cdot (19.56 - x)$$

Solving,  $x = 3.72 \text{ in}$

$$\text{Then } j \cdot d_s = d_s - \frac{x}{3} = 19.56 - \frac{3.72}{3} = 18.32 \text{ in}$$

$$s = 10 \text{ in} \leq \frac{700 \cdot \gamma_e}{\beta_s \cdot \frac{M_{\text{drop}}}{A_s \cdot j \cdot d_s}} - 2 \cdot d_c$$

Solving for  $M_{\text{drop}}$ :

$$\begin{aligned} M_{\text{drop}} &= \frac{700 \cdot \gamma_e \cdot A_s \cdot j \cdot d_s}{\beta_s \cdot (s + 2 \cdot d_c)} = \frac{700 \cdot 0.75 \cdot 0.72 \cdot 18.32}{1.14 \cdot (10 + 2 \cdot 1.94)} \cdot \frac{1}{12} \\ &= 36.5 \text{ kip-ft} \end{aligned}$$

Interpolate to determine span point location of drop point:

For Span 1:

$$1.7 + \left( \frac{38 - 36.5}{38 - 11} \right) \cdot 0.1 = 1.71 \text{ or } 10.44 \text{ ft from Pier 1 centerline.}$$

For Span 2:

$$2.2 + \left( \frac{36.5 - 21}{49 - 21} \right) \cdot 0.1 = 2.26 \text{ or } 11.70 \text{ ft from Pier 1 centerline.}$$

Use the drop point locations based on crack control.

By inspection, the fatigue stress range check and the minimum reinforcement check are satisfied.

**[5.10.8.1.2a]**

Due to the uncertainty associated with the design moments, the reinforcement cannot be terminated at the theoretical drop point. It must be carried beyond the theoretical point by the greatest of: the effective depth of the member, 15 times the nominal diameter of the bar, or  $1/20$  of the clear span.

The required extension  $L_{\text{ext1}}$  for Span 1 is:

$$L_{\text{ext1}} = d_s = 19.56 \text{ in}$$

or

$$L_{\text{ext1}} = 15 \cdot d_b = 15 \cdot 0.875 = 13.125 \text{ in}$$

or

$$L_{\text{ext1}} = \frac{1}{20} \cdot (36 \cdot 12) = 21.6 \text{ in} \quad \text{GOVERNS}$$

The required extension  $L_{\text{ext2}}$  for Span 2 is:

$$L_{\text{ext2}} = d_s = 19.56 \text{ in}$$

or

$$L_{\text{ext2}} = 15 \cdot d_b = 15 \cdot 0.875 = 13.125 \text{ in}$$

or

$$L_{\text{ext2}} = \frac{1}{20} \cdot (45 \cdot 12) = 27.0 \text{ in} \quad \text{GOVERNS}$$

Adding the extension length to the theoretical distance from the pier at which the bars can be dropped results in the following cutoff locations for the interior strip:

$$\text{For Span 1: } 10.44 - \frac{21.6}{12} = 8.64 \text{ ft} \quad \text{Use } 8'-6"$$

$$\text{For Span 2: } 11.70 - \frac{27.0}{12} = 9.45 \text{ ft} \quad \text{Use } 9'-0"$$

For the exterior strip, the positive bending moments are:

Span Point	M <sub>Strength I</sub> (kip-ft)/ft	M <sub>Service I</sub> (kip-ft)/ft
1.7	81	48
1.8	33	15
2.2	50	28
2.3	102	63

Similarly for the exterior strip:

The moment capacity of #8 bars at 10 inches ( $A_s = 0.95 \text{ in}^2$ ) for positive flexure  $M_r$  is 80.4 kip-ft.

For Span 1, the drop point location based on strength is 1.70 or 10.80 ft from Pier 1 centerline.

For Span 2, the drop point location based on strength is 2.26 or 11.70 ft from Pier 1 centerline.

In order to meet the crack control requirements, the Service I positive moment at the drop point,  $M_{\text{drop}} = 46.9 \text{ kip-ft}$ .

For Span 1, the drop point location based on crack control is 1.70 or 10.80 ft from Pier 1 centerline.

For Span 2, the drop point location based on crack control is 2.25 or 11.25 ft from Pier 1 centerline.

Use the drop point locations based on crack control.

Adding the extension length to the theoretical distance from the pier at which the bars can be dropped results in the following cutoff locations for the exterior strip:

$$\text{For Span 1: } 10.80 - \frac{21.6}{12} = 9.00 \text{ ft} \quad \text{Use 9'-0"}$$

$$\text{For Span 2: } 11.25 - \frac{27.0}{12} = 9.00 \text{ ft} \quad \text{Use 9'-0"}$$

#### [5.10.8.1.2b]

By continuing half of the reinforcement for the entire length of the bridge, LRFD Article 5.10.8.1.2b is satisfied.

#### [5.7.3.5]

#### Check Longitudinal Reinforcement

Check the minimum longitudinal reinforcement requirements at the abutments, assuming that a diagonal crack would start at the inside edge of the bearing area.

The slab sits on a 3'-0" wide integral abutment.

For  $\theta = 45^\circ$  determine the length from the end of the slab,  $L_{\text{crack}}$ , at which a diagonal crack will intersect the bottom longitudinal reinforcement (#7 bars @ 5"):

$$L_{\text{crack}} = 36.00 + 1.94 \cdot \cot(45^\circ) = 37.94 \text{ in}$$

From Figure 5.2.2.2 of this manual, the development length for #7 epoxy coated bars @ 5" with 1.5" cover and  $\leq 12''$  concrete cast below is:

$$\ell_{d7} = 3'-7'' = 43''$$

Then the tensile resistance of the longitudinal bars at the crack location

$$\begin{aligned} T_r &= f_y \cdot A_s \cdot \frac{L_{\text{crack}} - (\text{end cover})}{\ell_{d7}} \\ &= 60 \cdot 1.44 \cdot \left( \frac{37.94 - (\sim 3.5)}{43.0} \right) = 69.2 \text{ kips} \end{aligned}$$

The force to be resisted is:

$$\begin{aligned} T_u &= \left( \frac{V_u}{\phi} - 0.5 V_s - V_p \right) \cot \theta \\ &= \left( \frac{13.7}{0.9} - 0.5 \cdot 0 - 0 \right) \cot 45^\circ \\ &= 15.2 \text{ kips} < 69.2 \text{ kips} \end{aligned}$$

OK

Note that LRFD C5.7.3.5 states that  $V_u$  may be taken at  $d_v$  away from the face of support. For simplicity, the value for  $V_u$  at the abutment centerline of bearing was used in the equation above.

Similarly for the exterior strip:

$$\begin{aligned} L_{\text{crack}} &= 38.00 \text{ in} \\ \ell_{d8} &= 4' - 6'' = 54'' \end{aligned}$$

$$T_r = 72.8 \text{ kips}$$

$$T_u = 15.2 \text{ kips} < 72.8 \text{ kips}$$

OK

**L. Design Negative  
Moment  
Reinforcement**  
[5.6.2.2]  
[5.6.3.2]

Determine the required area of flexural reinforcement to satisfy the Strength I Load Combination.

**Flexural Resistance**

Assume a rectangular stress distribution and solve for the required area of reinforcing based on the factored moment,  $M_u$ , and an assumed depth,  $d_s$ . Also assume the section is tension-controlled and the flexural resistance factor  $\phi$  is 0.90.



Use the same general equation developed for the positive moment reinforcement.

$$A_s = \frac{4.5 \cdot d_s - \sqrt{20.25 \cdot d_s^2 - 13.236 \cdot M_u}}{6.618}$$

For the calculation of  $d_s$ , assume #8 bars for the interior strip and #10 bars for the exterior strip.

$$d_{s\_int} = 22 - 3 - 0.5 \cdot (1.00) = 18.50 \text{ in}$$

$$d_{s\_ext} = 22 - 3 - 0.5 \cdot (1.27) = 18.37 \text{ in}$$

The required area of steel and trial reinforcement is presented in Table 5.7.1.11. After evaluating the areas of steel required, a layout based on a bar spacing of 5 inches was selected.

As the negative moment reduces away from the supports, the required area of steel is reduced, so theoretically the bar area or spacing could change multiple times along the length of the bridge. Later in this example, using the guidance in BDM Article 5.3.4, we will design for one change located between the point of maximum negative moment at the pier and midspan of each adjacent span. Half of the top longitudinal bars will be dropped, resulting in a new bar spacing of 10 inches.

**Table 5.7.1.11**  
**Trial Top Longitudinal Reinforcement**

Span Point	Interior Strip					Exterior Strip				
	$M_u$ (k-ft)/ft	$d_s$ (in)	Reqd. $A_s$ (in <sup>2</sup> /ft)	Trial Bars	Provided $A_s$ (in <sup>2</sup> /ft)	$M_u$ (k-ft)/ft	$d_s$ (in)	Reqd. $A_s$ (in <sup>2</sup> /ft)	Trial Bars	Provided $A_s$ (in <sup>2</sup> /ft)
2.0	-144	18.50	1.87	#8 @ 5	1.90	-181	18.37	2.43	#10 @ 5	3.05

Similar to the positive moment sections, validate the assumption of 0.9 for resistance factor for the interior strip:

Calculate the depth of the Whitney stress block.

$$a = \frac{A_s \cdot f_y}{\phi_1 \cdot f'_c \cdot b} = \frac{1.90 \cdot 60}{0.85 \cdot 4 \cdot 12} = 2.79 \text{ in}$$

The depth of the section in compression is:

$$c = \frac{a}{\beta_1} = \frac{2.79}{0.85} = 3.28 \text{ in}$$

Concrete compression strain limit  $\epsilon_c = 0.003$

Reinforcement tension-controlled strain limit  $\epsilon_{tI} = 0.005$

$$\epsilon_t = (d_s - c) \cdot \left( \frac{\epsilon_c}{c} \right) = (18.50 - 3.28) \cdot \left( \frac{0.003}{3.28} \right) = 0.0139 > 0.005$$

Therefore, the interior strip section is tension controlled and  $\phi = 0.90$ .

Similarly for the exterior strip:

$$a = 4.49 \text{ in}$$

$$c = 5.28 \text{ in}$$

$$\epsilon_t = 0.0074 > 0.005$$

Therefore, the exterior strip section is also tension controlled and  $\phi = 0.90$ .

### [5.6.7]

#### Crack Control

At Span Point 2.0 of the interior strip, the Service I moment is -97 kip-ft

At Span Point 2.0 of the exterior strip, the Service I moment is -122 kip-ft

Similar to the positive moment sections, the stress in the reinforcement is found using a cracked section analysis with the trial reinforcement. For this check, the section is assumed to be singly reinforced.

Referring to Figure 5.7.1.3, determine the location of the neutral axis for the interior strip:

$$b \cdot x \cdot \frac{x}{2} = n \cdot A_s \cdot (d_s - x)$$

$$\frac{(12) \cdot x^2}{2} = 7.27 \cdot 1.90 \cdot (18.50 - x)$$

$$\text{Solving, } x = 5.48 \text{ in}$$

Determine the lever arm between service load flexural force components.

$$j \cdot d_s = d_s - \frac{x}{3} = 18.50 - \frac{5.48}{3} = 16.67 \text{ in}$$

The maximum allowable tension stress  $f_{ssmax}$  is 36.0 ksi

Compute the actual stress in the reinforcement.

$$f_{ss} = \frac{M}{A_s \cdot j \cdot d_s} = \frac{97 \cdot 12}{1.90 \cdot (16.67)} = 36.8 \text{ ksi} > 36.0 \text{ ksi} \quad \text{NO GOOD}$$

### Revise Interior Strip Top Longitudinal Bars

At this point, we can increase the bar size, change the bar spacing, or both. In order to keep the spacings consistent, try:

$$\#9 \text{ bars @ } 5'' \text{ spacing, } A_s = 2.40 \text{ in}^2/\text{ft}$$

Recalculate flexural resistance:

$$d_s = 18.44 \text{ in}$$

$$a = 3.53 \text{ in}$$

$$M_r = 180.1 \text{ k-ft} > 144 \text{ k-ft} \quad \text{OK}$$

$$c = 4.15 \text{ in}$$

$$\epsilon_t = 0.0103 > 0.005$$

Recalculate crack control checks:

$$x = 6.01 \text{ in}$$

$$j \cdot d_s = 16.44 \text{ in}$$

$$f_{ss} = 29.5 \text{ ksi} < 36.0 \text{ ksi} \quad \text{OK}$$

Actual cover  $d_{covtop} = 3.0 \text{ in}$

### [BDM 5.3.2]

For the calculation of  $d_c$ , use a maximum clear cover  $d_{covtopcc}$  equal to 2.0 inches.

$$\text{Then } d_c = d_{covtopcc} + 0.5 \cdot d_b = 2.0 + 0.5 \cdot 1.128 = 2.56 \text{ in}$$

$$\beta_s = 1 + \frac{d_c}{0.7 \cdot (h - h_{wear} - d_c)} = 1 + \frac{2.56}{0.7 \cdot (22 - 0.5 - 2.56)} = 1.19$$

Use  $\gamma_e = 0.75$ :

$$s \leq \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = \frac{700 \cdot 0.75}{1.19 \cdot 29.5} - 2 \cdot 2.56 = 9.8 \text{ in} > 5 \text{ in} \quad \text{OK}$$

Similarly, for the exterior strip:

$$x = 6.60 \text{ in}$$

$$j \cdot d_s = 16.17 \text{ in}$$

$$f_{ss} = 29.7 \text{ ksi} < 36.0 \text{ ksi} \quad \text{OK}$$

$$d_c = 2.64 \text{ in}$$

$$\beta_s = 1.19$$

$$s_{\max} = 9.6 \text{ in} > 5 \text{ in}$$

OK

**[5.5.3]****Fatigue**

The stress range in the reinforcement is computed and compared against code limits to ensure adequate fatigue resistance is provided.

**[Table 3.4.1-1]**

Fatigue I:  $U = 1.0 \cdot 1.75$  (LL + IM)

**[3.6.2.1]**

The dynamic load allowance for fatigue,  $IM = 15\%$

At Span Point 2.0 the one lane fatigue moments are:

Maximum positive moment = 33 kip-ft

Maximum negative moment = -252 kip-ft

Multiply the one lane moments by the dynamic load allowance, and the fatigue live load distribution factor to get the fatigue moments for a 1 foot wide strip. Note that negative moment causes tension in the top bars, but the sign convention for fatigue checks is + for tension and - for compression, so switch the signs on the moments to do this calculation:

$$\text{Fatigue LL } M_{\text{fatmin}} = -(33) \cdot 1.15 \cdot 0.058 = -2.2 \text{ kip-ft}$$

$$\text{Fatigue LL } M_{\text{fatmax}} = -(-252) \cdot 1.15 \cdot 0.058 = 16.8 \text{ kip-ft}$$

**[5.5.3.1]**

AASHTO allows the use of uncracked section properties where the sum of the unfactored dead load stress and Fatigue I stress is less than  $0.095\sqrt{f'_c}$  in tension. Conservatively assume the section is cracked.

Then using the equation previously used to check crack control stresses, determine the fatigue live load stress range  $\Delta f$ :

$$\Delta f = \frac{M_{\text{fatmax}} - M_{\text{fatmin}}}{A_s \cdot j \cdot d_s} = \frac{[16.8 - (-2.2)] \cdot 12}{2.40 \cdot 16.44} = 5.8 \text{ ksi}$$

For the Fatigue I load combination, the factored live load stress is:

$$\gamma \cdot \Delta f = 1.75 \cdot 5.8 = 10.2 \text{ ksi}$$

**[5.5.3.2]**

Now calculate the constant-amplitude fatigue threshold  $(\Delta F)_{\text{TH}}$ . The unfactored dead load moment at Span Point 2.0 is -52.4 kip-ft. Again, we will switch the sign on  $M_{\text{DC}}$  to match the sign convention for fatigue checks (+ for tension, - for compression):

$$M_{\text{DC}} = -(-52.4) = 52.4 \text{ kip-ft}$$

$$f_{\min} = \frac{\gamma \cdot M_{\text{fatmin}} + M_{\text{DC}}}{A_s \cdot j \cdot d_s} = \frac{[1.75 \cdot (-2.2) + 52.4] \cdot 12}{2.40 \cdot 16.44} = 14.8 \text{ ksi}$$

$$(\Delta F)_{\text{TH}} = 26 - \frac{22 \cdot f_{\min}}{f_y} = 26 - \frac{22 \cdot (14.8)}{60} = 20.6 \text{ ksi} > 10.2 \text{ ksi} \quad \text{OK}$$

**[5.6.3.3]****Check Minimum Reinforcement**

To prevent a brittle failure, adequate flexural reinforcement needs to be placed in the cross section. For this check, zero wear is conservatively assumed.

Check that the reinforcement can carry the smaller of:

- Cracking moment,  $M_{\text{cr}}$
- $1.33 \cdot M_u$

At Span Point 2.0:

$$1.33 \cdot M_u = 1.33 \cdot 144 = 191.5 \text{ kip-ft}$$

$$f_r = 0.24 \cdot \lambda \cdot \sqrt{f'_c} = 0.24 \cdot 1.0 \cdot \sqrt{4.0} = 0.48 \text{ ksi}$$

$$y_t = \frac{h}{2} = \frac{22.0}{2} = 11.0 \text{ in}$$

$$I_g = \frac{1}{12} \cdot b \cdot h^3 = \frac{1}{12} \cdot 12 \cdot (22)^3 = 10,648 \text{ in}^4$$

$$S_c = \frac{I_g}{y_t} = \frac{10648}{11.0} = 968 \text{ in}^3$$

For non-precast segmental structures,  $\gamma_1 = 1.6$

MnDOT uses AASHTO M31 (ASTM A615) Grade 60 reinforcement in concrete bridge structures, so  $\gamma_3 = 0.67$

$$M_{\text{cr}} = \gamma_3 \cdot (\gamma_1 \cdot f_r \cdot S_c) = \frac{0.67 \cdot (1.6 \cdot 0.48 \cdot 968)}{12} = 41.5 \text{ kip-ft} \quad \text{GOVERNS}$$

$$M_r = \phi \cdot A_s \cdot f_y \cdot \left(d_s - \frac{a}{2}\right)$$

$$M_r = 0.9 \cdot (2.40) \cdot (60) \cdot \left(18.44 - \frac{3.53}{2}\right) \cdot \frac{1}{12}$$

$$= 180.1 \text{ kip-ft} > M_{\text{cr}} = 41.5 \text{ kip-ft}$$

OK

Use #9 bars at 5 inches at Span Point 2.0

Similarly for the exterior strip:

$$\text{At Span Point 2.0, } 1.33 \cdot M_u = 240.7 \text{ kip-ft}$$

$$M_{\text{cr}} = 41.5 \text{ kip-ft}$$

$$M_r = 221.3 \text{ kip-ft} > 41.5 \text{ kip-ft}$$

Use #10 bars at 5 inches at Span Point 2.0

**[5.10.8.1.2a]**

**[5.10.8.1.2c]**

**Bar Cutoff Location**

Determine the location where the 5 inch spacing can be increased to 10 inches. The moment capacity of #9 bars at 10 inches ( $A_s = 1.20 \text{ in}^2$ ) for negative flexure is:

$$\begin{aligned} M_r &= \phi \cdot A_s \cdot f_y \cdot \left( d_s - \frac{a}{2} \right) \\ &= 0.9 \cdot (1.20) \cdot (60) \cdot \left[ 18.44 - \frac{1.20 \cdot (60)}{2 \cdot (0.85) \cdot (4) \cdot (12)} \right] \cdot \frac{1}{12} \\ &= 94.8 \text{ kip-ft} \end{aligned}$$

For the interior strip, the negative bending moments are:

Span Point	M <sub>Strength I</sub> (kip-ft)/ft	M <sub>Service I</sub> (kip-ft)/ft
1.8	-65	-40
1.9	-98	-64
2.0	-144	-97
2.1	-79	-52
2.2	-41	-24

Knowing that span points are 3.6 feet apart in Span 1 and 4.5 feet apart in Span 2, the drop point locations which meet the Strength I negative bending moment of 94.8 kip-ft can be found.

For Span 1, interpolate between Span Points 1.8 and 1.9:

$$1.8 + \left( \frac{94.8 - 65}{98 - 65} \right) \cdot 0.1 = 1.89 \text{ or } 3.96 \text{ ft from Pier 1 centerline.}$$

For Span 2, interpolate between Span Points 2.0 and 2.1:

$$2.0 + \left( \frac{144 - 94.8}{144 - 79} \right) \cdot 0.1 = 2.08 \text{ or } 3.60 \text{ ft from Pier 1 centerline.}$$

The reinforcement must also meet the serviceability requirements at the theoretical drop point. Determine the drop point location based on the crack control requirements and compare with the drop points based on strength to see which ones govern.

**[5.6.7]**

For #9 bars @ 10", ( $A_s = 1.20 \text{ in}^2$ ),  $d_c = 2.56 \text{ in}$ :

Determine neutral axis:

$$n \cdot A_s = 7.27 \cdot 1.20 = 8.72 \text{ in}^2$$

$$b \cdot x \cdot \frac{x}{2} = n \cdot A_s \cdot (d_s - x)$$

$$\frac{12 \cdot x^2}{2} = 8.72 \cdot (18.44 - x)$$

$$\text{solving, } x = 4.50 \text{ in}$$

$$\text{Then } j \cdot d_s = d_s - \frac{x}{3} = 18.44 - \frac{4.50}{3} = 16.94 \text{ in}$$

$$s = 10 \text{ in} \leq \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

Solve for the moment at the drop point:

$$\begin{aligned} M_{\text{drop}} &= \frac{700 \cdot \gamma_e \cdot A_s \cdot j \cdot d_s}{\beta_s \cdot (s + 2 \cdot d_c)} = \frac{700 \cdot 0.75 \cdot 1.20 \cdot 16.94}{1.19 \cdot (10 + 2 \cdot 2.56)} \cdot \frac{1}{12} \\ &= 49.4 \text{ kip-ft} \end{aligned}$$

Interpolate to determine span point location of drop point:

For Span 1:

$$1.8 + \left( \frac{49.4 - 40}{64 - 40} \right) \cdot 0.1 = 1.84 \text{ or } 5.76 \text{ ft from Pier 1 centerline.}$$

For Span 2:

$$2.1 + \left( \frac{52 - 49.4}{52 - 24} \right) \cdot 0.1 = 2.11 \text{ or } 4.95 \text{ ft from Pier 1 centerline.}$$

Use the drop point locations based on crack control.

By inspection, the fatigue stress range check and the minimum reinforcement check are satisfied.

#### [5.10.8.1.2a]

Due to the uncertainty associated with the design moments, the reinforcement cannot be terminated at the theoretical drop point. It must be carried beyond the theoretical point by the greater of: the depth of the member, 15 times the nominal diameter of the bar, or  $1/20$  of the clear span.

The required extension  $L_{\text{ext}1}$  for Span 1 is:

$$L_{\text{ext}1} = d_s = 18.44 \text{ in}$$

or

$$L_{\text{ext1}} = 15 \cdot d_b = 15 \cdot 1.128 = 16.9 \text{ in}$$

or

$$L_{\text{ext1}} = \frac{1}{20} \cdot (36 \cdot 12) = 21.6 \text{ in} \quad \text{GOVERNS}$$

The required extension  $L_{\text{ext2}}$  for Span 2 is:

$$L_{\text{ext2}} = \frac{1}{20} \cdot (45 \cdot 12) = 27.0 \text{ in}$$

Adding the extension length to the theoretical distance from the pier at which the bars can be dropped results in the following cutoff locations for the interior strip:

$$\text{For Span 1: } 5.76 + \frac{21.6}{12} = 7.56 \text{ ft} \quad \text{Use 8'-0"}$$

$$\text{For Span 2: } 4.95 + \frac{27.0}{12} = 7.20 \text{ ft} \quad \text{Use 7'-6"}$$

For the exterior strip, the negative bending moments are:

Span Point	$M_{\text{Strength I}}$ (kip-ft)/ft	$M_{\text{Service I}}$ (kip-ft)/ft
1.8	-83	-51
1.9	-125	-81
2.0	-181	-122
2.1	-100	-65
2.2	-54	-31

Similarly for the exterior strip:

The moment capacity of #10 bars at 10 inches ( $A_s = 1.52 \text{ in}^2$ ) for negative flexure  $M_r$  is 118.0 kip-ft

For Span 1, the drop point location based on strength is 1.88 or 4.32 ft from Pier 1 centerline.

For Span 2, the drop point location based on strength is 2.08 or 3.60 ft from Pier 1 centerline.

In order to meet the crack control requirements, the Service I negative moment at the drop point,  $M_{\text{drop}} = 61.1 \text{ kip-ft}$



For Span 1, the drop point location based on crack control is 1.83 or 6.12 ft from Pier 1 centerline.

For Span 2, the drop point location based on crack control is 2.11 or 4.95 ft from Pier 1 centerline.

Use the drop point locations based on crack control.

Adding the extension length to the theoretical distance from the pier at which the bars can be dropped results in the following cutoff locations for the exterior strip:

$$\text{For Span 1: } 6.12 + \frac{21.6}{12} = 7.92 \text{ ft} \quad \text{Use 8'-0"}$$

$$\text{For Span 2: } 4.95 + \frac{27.0}{12} = 7.20 \text{ ft} \quad \text{Use 7'-6"}$$

**[5.10.8.1.2c]**

By continuing half of the reinforcement for the entire length of the bridge, LRFD Article 5.10.8.1.2c is satisfied.

**M. Distribution Reinforcement**  
**[5.12.2.1]**

The amount of bottom transverse distribution reinforcement,  $A_{sdist}$ , may be taken as a percentage of the main reinforcement required:

$$PCT_{dist} = \frac{100}{\sqrt{L}} \leq 50 \%$$

Use Span 1,  $L = 36.0$  since this will result in the largest  $A_{sdist}$

$$PCT_{dist} = \frac{100}{\sqrt{36}} = 16.7 \%$$

**[BDM 5.3.2]**

The maximum amount of bottom longitudinal reinforcement is found in the exterior strip (#8 bars at 5 inches). However, use the interior strip (#7 bars at 5 inches,  $A_s = 1.44 \text{ in}^2/\text{ft}$ ) as the basis for the distribution reinforcement per BDM Article 5.3.2. Then the required transverse reinforcement,  $A_{sdist}$ , is:

$$A_{sdist} = 0.167 \cdot 1.44 = 0.24 \text{ in}^2/\text{ft}$$

Use #5 @ 12",  $A_s = 0.31 \text{ in}^2/\text{ft}$  for bottom transverse reinforcement.

**N. Shrinkage and Temperature Reinforcement**  
**[5.10.6]**

Adequate reinforcement needs to be provided in the slab to ensure that cracks from shrinkage and temperature changes are small and well distributed.

$$\text{Temperature } A_s \geq \frac{1.30 \cdot b \cdot h}{2 \cdot (b + h) \cdot f_y} = \frac{1.30 \cdot 564 \cdot 22}{2 \cdot (564 + 22) \cdot 60} = 0.23 \text{ in}^2/\text{ft}$$

$$\text{Also, } 0.11 \leq A_s \leq 0.60$$

Try #5 @ 12",  $A_s = 0.31 \text{ in}^2/\text{ft}$  for top transverse reinforcement.

**O. Structural  
Analysis of Slab  
Edge Region  
[A13.4.1]**

The slab top transverse reinforcement must be checked to see if it is adequate to resist loads transferred from the bridge railing due to a vehicle collision. The Extreme Event II limit state is checked at the toe of the barrier for the dead load plus the horizontal collision force. Refer to Memo to Designers #2020-01 for the MnDOT requirements for this check.

**Geometry and Loads**

Referring to Figure 5.7.1.3, determine the center of gravity location for the barrier by considering the area of a rectangular block that encompasses the entire barrier cross-section and subtracting components ① and ②. Results are shown in Table 5.7.1.12:

**Table 5.7.1.12 Determination of Barrier Center of Gravity Location**

Component Description	Width (in)	Height (in)	Area (in <sup>2</sup> )	Moment Arm From Barrier Toe (in)	Area · Moment Arm (in <sup>3</sup> )
Block encompassing barrier	18.00	36.00	648.00	9.00	5832.00
① (triangle)	7.00	36.00	-126.00	2.33	-293.58
② (rectangle)	2.00	23.00	-46.00	17.00	-782.00

$$\text{Total Area, } A_{\text{total}} = 476.00 \text{ in}^2$$

$$\text{Total (Area · Moment Arm), } A_{\text{area} \cdot \text{M}_{\text{arm}}} = 4756.42 \text{ in}^3$$

$$\text{Then C.G. location, } X_{\text{cg}}, \text{ from barrier toe is: } X_{\text{cg}} = \frac{4756.42}{476.00} = 9.99 \text{ in}$$

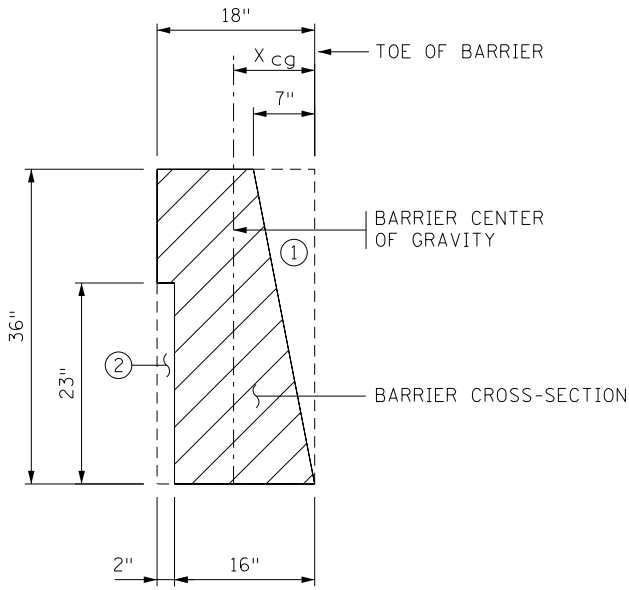
Referring to Figure 5.7.1.4, determine the dead load moments acting on the slab at the toe of barrier for a 1 ft slab strip width. Results are shown in Table 5.7.1.13:

**Table 5.7.1.13 Determination of Dead Load Moments at Barrier Toe**

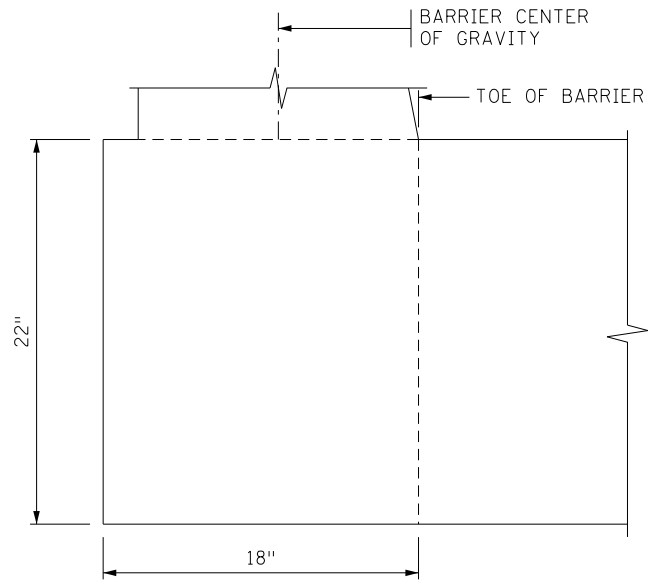
Component Description	Width (in)	Height (in)	$w_{\text{DC}}$ (kips)	Moment Arm From Barrier Toe (in)	Unfactored Moment $M_{\text{DC}}$ (kip-ft)
Barrier			0.496	9.99	0.413
Slab	18.00	22.00	0.413	9.00	0.310

$$\text{Total } w_{\text{DC}} = 0.909 \text{ kips for a 1 ft strip width}$$

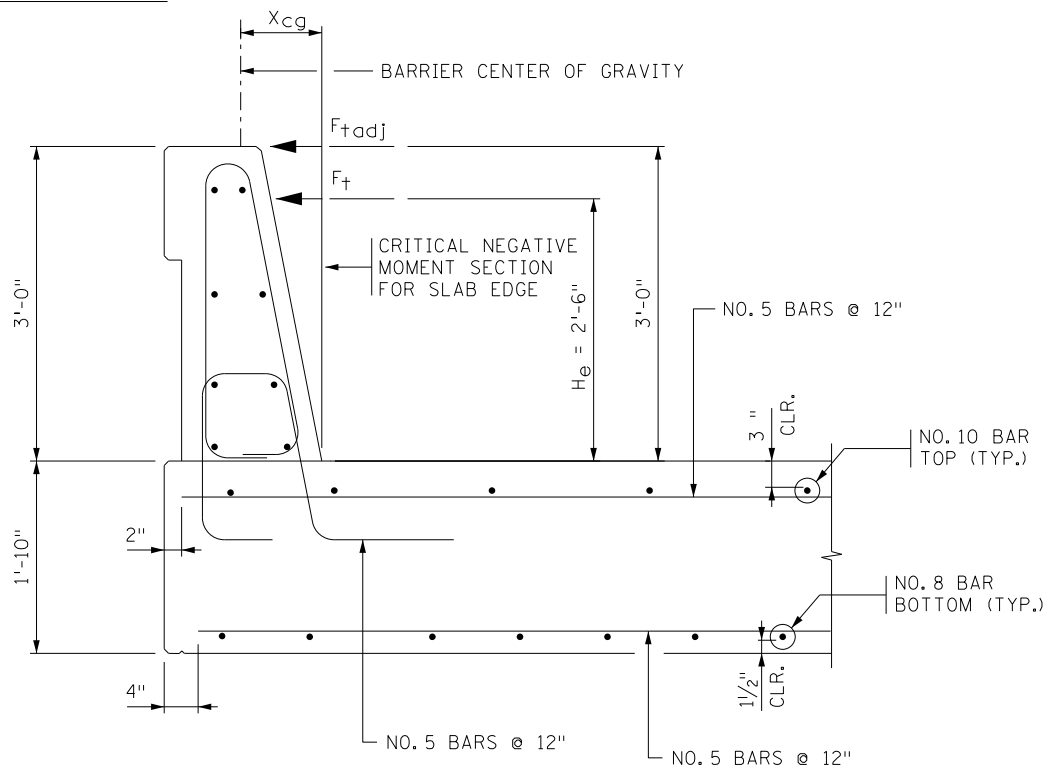
$$\text{Total } M_{\text{DC}} = 0.723 \text{ kip-ft for a 1 ft strip width}$$



BARRIER CENTER OF GRAVITY DETERMINATION



SLAB EDGE DEAD LOAD DETERMINATION



CROSS-SECTION AT SLAB EDGE

**Figure 5.7.1.4**

**[A13.2]****Collision Force Tension and Bending Moment**

Using the yield line analysis method of LRFD Appendix A13, values for the nominal resistance,  $R_w$ , flexural resistance about the horizontal axis,  $M_c$ , and critical wall length,  $L_c$ , were calculated for a 36 inch Type S barrier mounted on a slab without a wearing course (Std. Figure 5-397.138(A)) and listed in Table 1 of Memo to Designers #2020-01:

$$\text{Barrier int.: } R_{w\_int} = 119.9 \text{ kips} \quad M_{c\_int} = 17.0 \text{ kip-ft/ft} \quad L_{c\_int} = 10.6 \text{ ft}$$

$$\text{Barrier int.: } R_{w\_end} = 75.3 \text{ kips} \quad M_{c\_end} = 22.9 \text{ kip-ft/ft} \quad L_{c\_end} = 4.9 \text{ ft}$$

For a barrier meeting MASH Test Level 4:

$$\text{Transverse collision load } F_t = 80 \text{ kips}$$

$$\text{Height of load application } H_e = 30 \text{ in}$$

For this example, the ends of the bridge are supported by integral abutments. Therefore, the edges of the slab are fully supported at the bridge ends, so a collision load applied to the barrier at the abutment will not result in substantial loading of the slab end edge region. This means the slab interior edge region governs and a check of the end edge region is not required.

Because the yield line equations in LRFD assume the collision load is applied at the top of the barrier, adjust  $F_t$  for the difference between the barrier height and height of application. Refer to Figure 5.7.1.4.

$$H_{\text{barrier}} = 36 \text{ in}$$

$$F_{\text{tadj}} = F_t \cdot \left( \frac{H_e}{H_{\text{barrier}}} \right) = 80 \cdot \left( \frac{30}{36} \right) = 66.7 \text{ kips}$$

MnDOT requires that the slab edge be designed to resist a transverse collision force equal to the lesser of the barrier capacity,  $R_w$ , or  $F_{\text{tadj}}$ :

$$F_{\text{coll\_int}} = R_{w\_int} = 119.9 \text{ kips}$$

or

$$F_{\text{coll}} = F_{\text{tadj}} = 66.7 \text{ kips}$$

GOVERNS

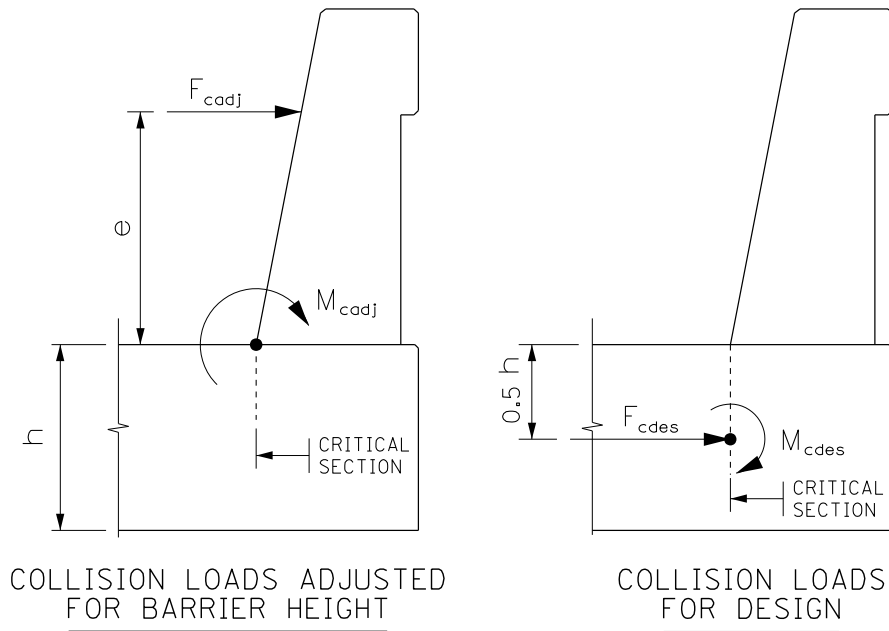
Since  $R_{w\_int}$  does not govern, the  $M_c$  value must also be adjusted to correspond with the collision load:

$$M_{\text{cadj\_int}} = \frac{F_{\text{coll\_int}}}{R_{w\_int}} \cdot M_{c\_int} = \frac{66.7}{119.9} \cdot 17.0 = 9.5 \frac{\text{kip-ft}}{\text{ft}}$$

For slab edge design, assume that the collision load is distributed over a length of  $L_{c\_int} + 2 \cdot H_{barrier}$  for the interior edge region:

$$F_{cadj\_int} = \frac{F_{coll\_int}}{L_{c\_int} + 2 \cdot H_{barrier}} = \frac{66.7}{10.6 + 2 \cdot \frac{36}{12}} = 4.0 \text{ kips/ft}$$

The resulting load  $M_{cadj}$  is located at the top of the slab at the toe of the barrier. Translate this load to the center of the slab for design of the slab edge. Referring to Figure 5.7.1.5, first find the eccentricity:



**Figure 5.7.1.5**

$$e_{int} = \frac{M_{cadj\_int}}{F_{cadj\_int}} = \frac{9.5}{4.0} = 2.38 \text{ ft}$$

Then:

$$F_{cdes\_int} = F_{cadj\_int} = 4.0 \text{ kips/ft}$$

$$M_{cdes\_int} = F_{cdes\_int} \cdot (e_{int} + 0.5 \cdot h)$$

$$= 4.0 \cdot \left( 2.38 + 0.5 \cdot \frac{22}{12} \right) = 13.2 \frac{\text{kip-ft}}{\text{ft}}$$

**[A13.4.1]****Extreme Event II Limit State Bending Moment**

Total factored loads are:

$$\begin{aligned} M_{u\_int} &= 1.00 \cdot M_{DC} + 1.00 \cdot M_{cdes\_int} \\ &= 1.00 \cdot 0.723 + 1.00 \cdot 13.2 = 13.9 \text{ kip-ft/ft} \end{aligned}$$

$$\begin{aligned} P_{u\_int} &= 1.00 \cdot F_{DC} + 1.00 \cdot F_{cdes\_int} \\ &= 1.00 \cdot 0.0 + 1.00 \cdot 4.0 = 4.0 \text{ kips/ft} \end{aligned}$$

The eccentricity of  $P_u$  is:

$$e_{u\_int} = \frac{M_{u\_int}}{P_{u\_int}} = \frac{13.9}{4.0} = 3.48 \text{ ft} = 41.76 \text{ in}$$

**Resistance of Slab Interior Edge Region**

The edge must resist both axial tension and bending moment. The capacity of the edge will be determined by considering the tension side of the structural interaction diagram for a one foot wide portion of the edge.

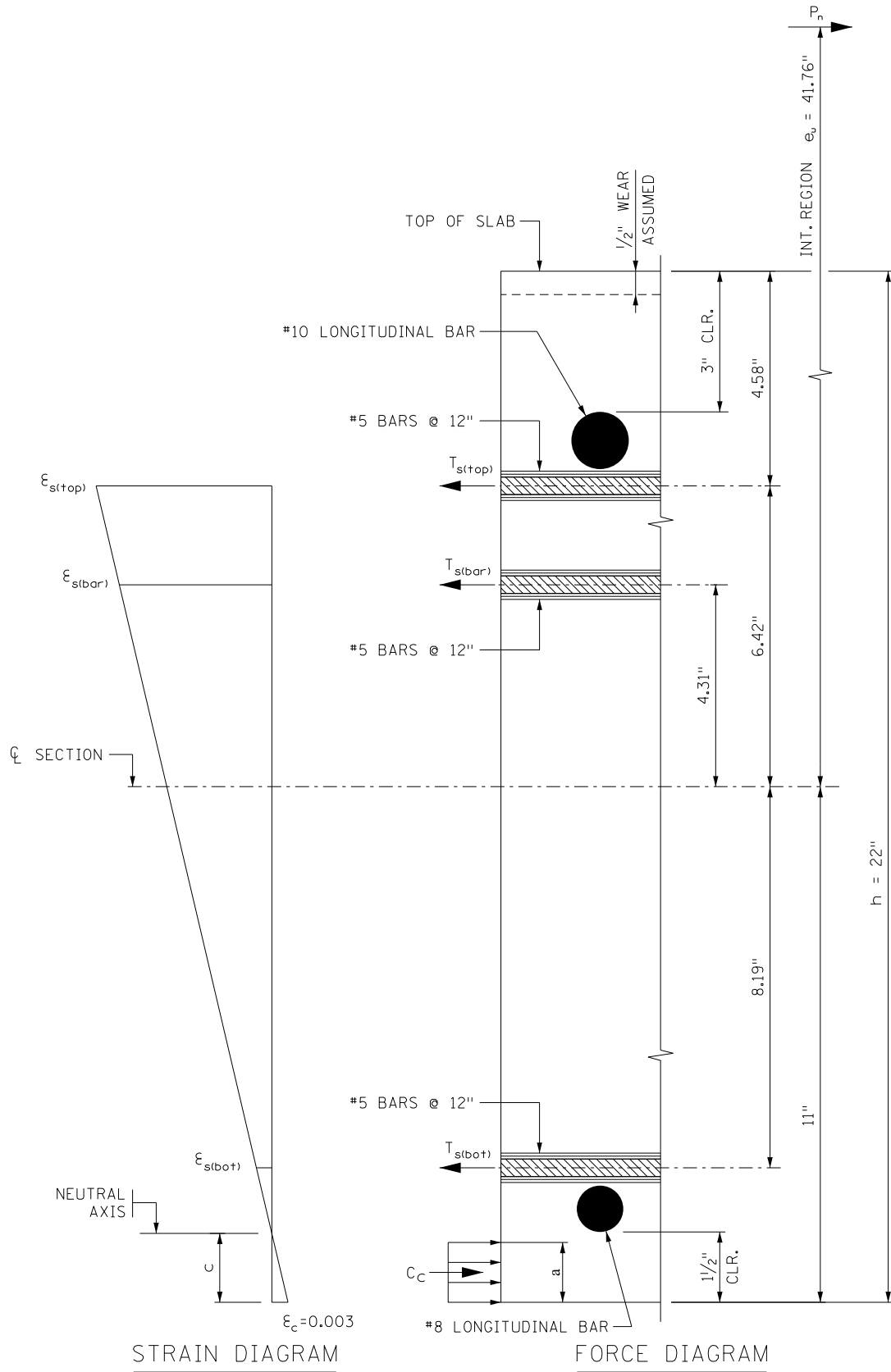
Check if the transverse reinforcement chosen previously for the slab based on shrinkage and temperature, and load distribution will be adequate:

$$\begin{aligned} \text{Top reinforcement} &- \#5 \text{ bars @ } 12'' \text{ (} A_{s(\text{top})} = 0.31 \text{ in}^2/\text{ft)} \\ \text{Bottom reinforcement} &- \#5 \text{ bars @ } 12'' \text{ (} A_{s(\text{bot})} = 0.31 \text{ in}^2/\text{ft)} \end{aligned}$$

Note that the front leg of the barrier bar also contributes to the strength of the edge. The barrier reinforcement is:

$$\text{Barrier front leg} - \#5 \text{ bars @ } 12'' \text{ (} A_{s(\text{bar})} = 0.31 \text{ in}^2/\text{ft)}$$

Referring to Figure 5.7.1.6, determine the capacity of the edge section for the eccentricity  $e_{u\_int}$  equal to 41.76 inches.



**Reinforced Concrete Section at Toe of Barrier**  
**Figure 5.7.1.6**

First, determine the effective area of reinforcement for the slab bars located at the toe of the barrier. From Figures 5.2.2.1 and 5.2.2.2 of this manual, the development length  $\ell_d$ , for the slab bars are as follows:

For #5 top bars @ 12", cover is 3.77" (assuming 0.5" of wear and #10 longitudinal bar in exterior strip) at inside of barrier toe, and more than 12" of concrete is cast below, which results in an  $\ell_d = 29"$ .

For #5 bottom bars @ 12", cover is 2.5" (based on #8 longitudinal bar in exterior strip) at inside of barrier toe, which results in an  $\ell_d = 22"$ .

For #5 barrier bars @ 12", cover is 6.5" to the top of slab (assuming 0.5" of wear) at the inside of the barrier toe, and more than 12" of concrete is cast below, which results in an  $\ell_d = 29"$ .

Referring to Figure 5.7.1.4, the distance from the edge of slab to the inside of the barrier toe,  $L_{crit}$ , is 18.00". Then:

$$\begin{aligned} \text{For \#5 top bars, } \ell_{d\text{available}} &= L_{crit} - (\text{top bar end cover}) \\ &= 18.00 - 2.0 = 16.00 \text{ in} \end{aligned}$$

$$A_{s(\text{top})\text{eff}} = A_{s(\text{top})} \cdot \frac{\ell_{d\text{available}}}{\ell_d} = 0.31 \cdot \frac{16.00}{29} = 0.17 \text{ in}^2/\text{ft}$$

$$\begin{aligned} \text{For \#5 bottom bars, } \ell_{d\text{available}} &= L_{crit} - (\text{bottom bar end cover}) \\ &= 18.00 - 4.0 = 14.00 \text{ in} \end{aligned}$$

$$A_{s(\text{bot})\text{eff}} = A_{s(\text{bot})} \cdot \frac{\ell_{d\text{available}}}{\ell_d} = 0.31 \cdot \frac{14.00}{22} = 0.20 \text{ in}^2/\text{ft}$$

For #5 barrier bars (R501E), the bar is considered fully developed on the outside of the barrier toe due to the bend. On the traffic side of the barrier toe, refer to Figure 5.7.1.7 to determine how much of the 18" bar leg extends beyond the toe.

$$\Delta_1 = \frac{2 + 0.5 \cdot (0.625)}{\cos 11^\circ} = 2.36 \text{ in}$$

$$\Delta_2 = 17 - 10 - 0.5 \cdot (0.625) = 6.69 \text{ in}$$

$$\Delta_3 = 6.69 \cdot (\tan 11^\circ) = 1.30 \text{ in}$$

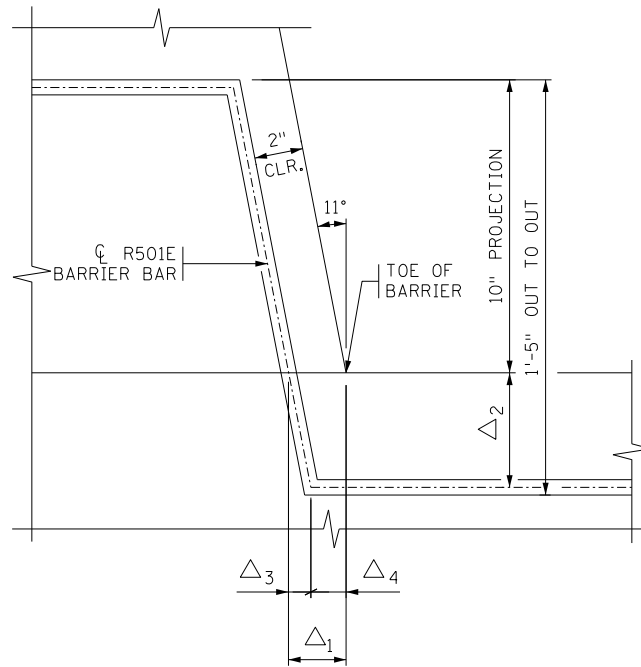
$$\Delta_4 = 2.36 - 1.30 = 1.06 \text{ in}$$

Then:

$$\ell_{d\text{available}} = 18 - 1.06 = 16.94 \text{ in}$$



$$A_{s(\text{bar})\text{eff}} = A_{s(\text{bar})} \cdot \frac{\ell_{\text{davailable}}}{\ell_d} = 0.31 \cdot \frac{16.94}{29} = 0.18 \text{ in}^2/\text{ft}$$



PARTIAL SECTION AT TOE OF BARRIER

**Figure 5.7.1.7**

Now determine the distance from the bottom of the section to the neutral axis,  $c$ . Start by assuming that for all reinforcement,  $\epsilon_s > \epsilon_y$ .

Then:

$$f_s = E_s \cdot \epsilon_y = f_y$$

$$T_{s(\text{top})} = A_{s(\text{top})\text{eff}} \cdot f_y = 0.17 \cdot 60 = 10.20 \text{ kips/ft}$$

$$T_{s(\text{bot})} = A_{s(\text{bot})\text{eff}} \cdot f_y = 0.20 \cdot 60 = 12.00 \text{ kips/ft}$$

$$T_{s(\text{bar})} = A_{s(\text{bar})\text{eff}} \cdot f_y = 0.18 \cdot 60 = 10.80 \text{ kips/ft}$$

$$T_{s(\text{tot})} = 10.20 + 12.00 + 10.80 = 33.00 \text{ kips/ft}$$

The total compression force  $C_c$  is:

$$C_c = 0.85 \cdot f_c \cdot b \cdot a = 0.85 \cdot 4.0 \cdot 12.0 \cdot 0.85 \cdot c = 34.68 \cdot c$$

Referring to Figure 5.7.1.6, find  $c$  by taking moments about  $P_n$ :

$$10.20 \cdot (41.76 - 6.42) + 12.00 \cdot (41.76 + 8.19) + 10.80 \cdot (41.76 - 4.31) - 34.68 \cdot c \cdot (41.76 + 11 - 0.5 \cdot 0.85 \cdot c) = 0$$

Solving, we get  $c = 0.75 \text{ in}$

Check if the original assumption was correct, that  $\epsilon_s > \epsilon_y$ :

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\epsilon_{s(\text{top})} = (22 - 4.58 - 0.75) \cdot \left(\frac{0.003}{0.75}\right) = 0.06668 > 0.00207$$

$$\epsilon_{s(\text{bot})} = (11 - 8.19 - 0.75) \cdot \left(\frac{0.003}{0.75}\right) = 0.00824 > 0.00207$$

$$\epsilon_{s(\text{bar})} = (11 + 4.31 - 0.75) \cdot \left(\frac{0.003}{0.75}\right) = 0.05824 > 0.00207$$

Therefore the assumption was correct.

Then,

$$C_c = 34.68 \cdot c = 34.68 \cdot 0.75 = 26.01 \text{ kips/ft}$$

And,

$$\begin{aligned} P_n &= T_{s(\text{top})} + T_{s(\text{bot})} + T_{s(\text{bar})} - C_c \\ &= 10.20 + 12.00 + 10.80 - 26.01 = 6.99 \text{ kips/ft} \end{aligned}$$

### [1.3.2.1]

The resistance factor  $\phi$  for Extreme Event II limit state is 1.0. Therefore,

$$\phi \cdot P_n = 1.0 \cdot 6.99 = 6.99 \text{ kips/ft} > 4.00 \text{ kips/ft} \quad \text{OK}$$

$$\begin{aligned} \phi \cdot M_n &= \phi \cdot P_n \cdot e_u \\ &= 1.0 \cdot 6.99 \cdot 41.76 \cdot \frac{1}{12} = 24.33 \frac{\text{kip-ft}}{\text{ft}} > 13.90 \frac{\text{kip-ft}}{\text{ft}} \quad \text{OK} \end{aligned}$$

Therefore, the slab interior edge region reinforcement is adequate.

### **P. Dead Load Camber**

#### [5.6.3.5.2]

#### [BDM 5.3.5]

The total weight of the superstructure is used for dead load deflections. The gross moment of inertia is used in a computer analysis to obtain instantaneous deflections presented in Table 5.7.1.8. A long-term deflection multiplier of 4.0 is used in conjunction with the gross moment of inertia. The slab is cambered upward an amount equal to the immediate deflection plus one half of the long-term deflection.

For the 36 ft side spans:

$$\text{Maximum instantaneous deflection, } \Delta_{\text{inst}13} = 0.105 \text{ in}$$

$$\text{Long-term deflection, } \Delta_{\text{term}13} = 4 \cdot (0.105) = 0.420 \text{ in}$$

$$\text{Upward camber, } \Delta_{\text{c}13} = 0.105 + \frac{0.420}{2} = 0.315 \text{ in}$$

Rounding to the nearest 1/8" gives an upward camber of 3/8" for the interior strip.

For the 45 ft center span:

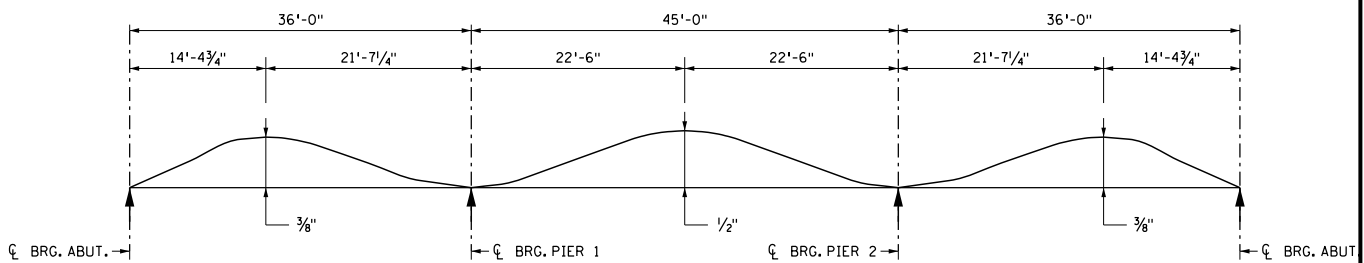
Maximum instantaneous deflection,  $\Delta_{inst2} = 0.136$  in

Long-term deflection,  $\Delta_{term2} = 4 \cdot (0.136) = 0.544$  in

Upward camber,  $\Delta_{c2} = 0.136 + \frac{0.544}{2} = 0.408$  in

Rounding to the nearest 1/8" gives an upward camber of 1/2" for the interior strip.

A camber diagram for the interior strip is shown in Figure 5.7.1.8 below:



**Dead Load Camber**  
**Figure 5.7.1.8**

**Q. Final Reinforcement Layout**

Figure 5.7.1.9 contains a plan view and Figure 5.7.1.10 contains a cross section that illustrates the reinforcement for the slab. The figures show that the exterior strips contain more reinforcing steel than the interior of the slab.

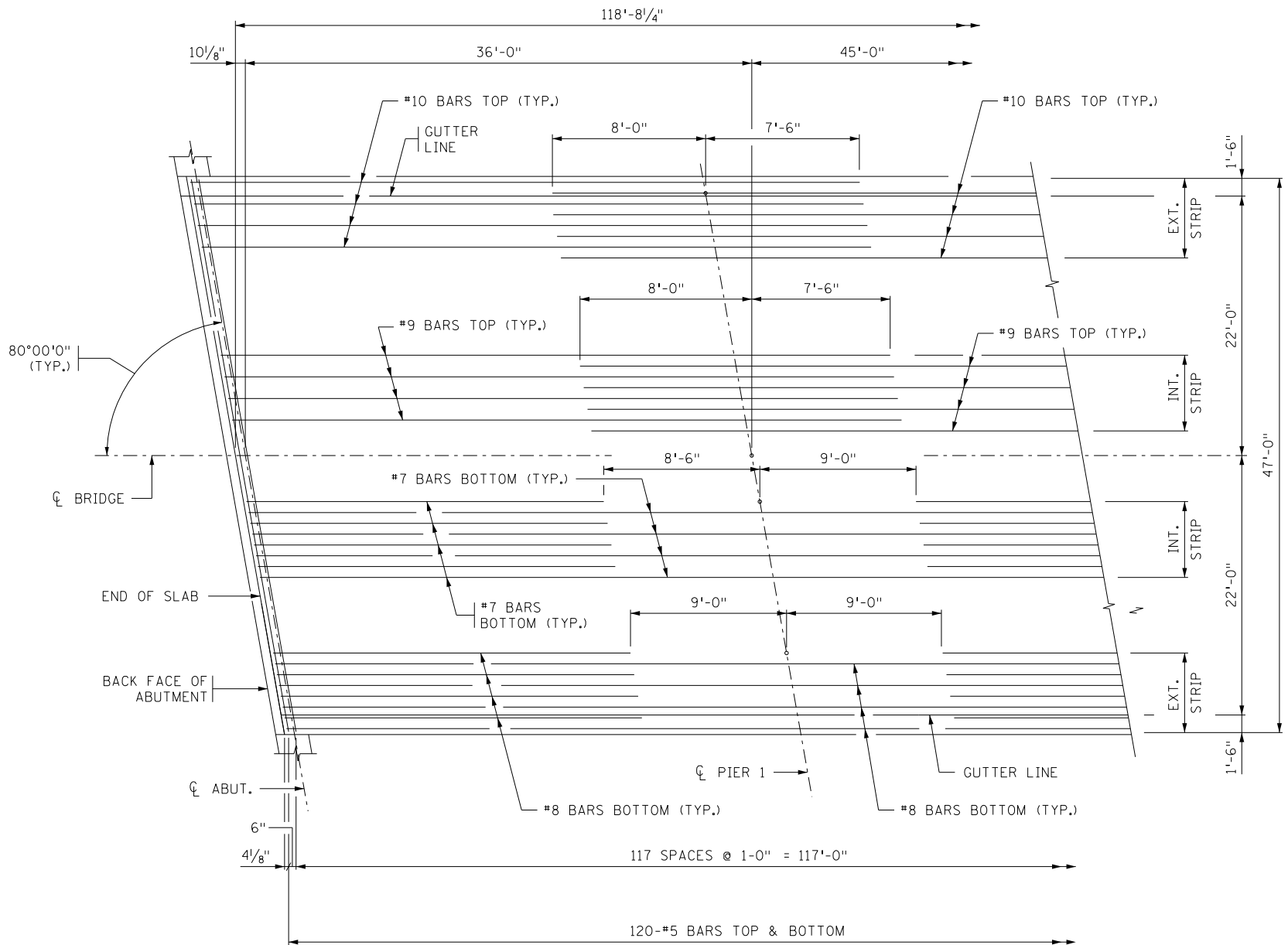


Figure 5.7.1.9

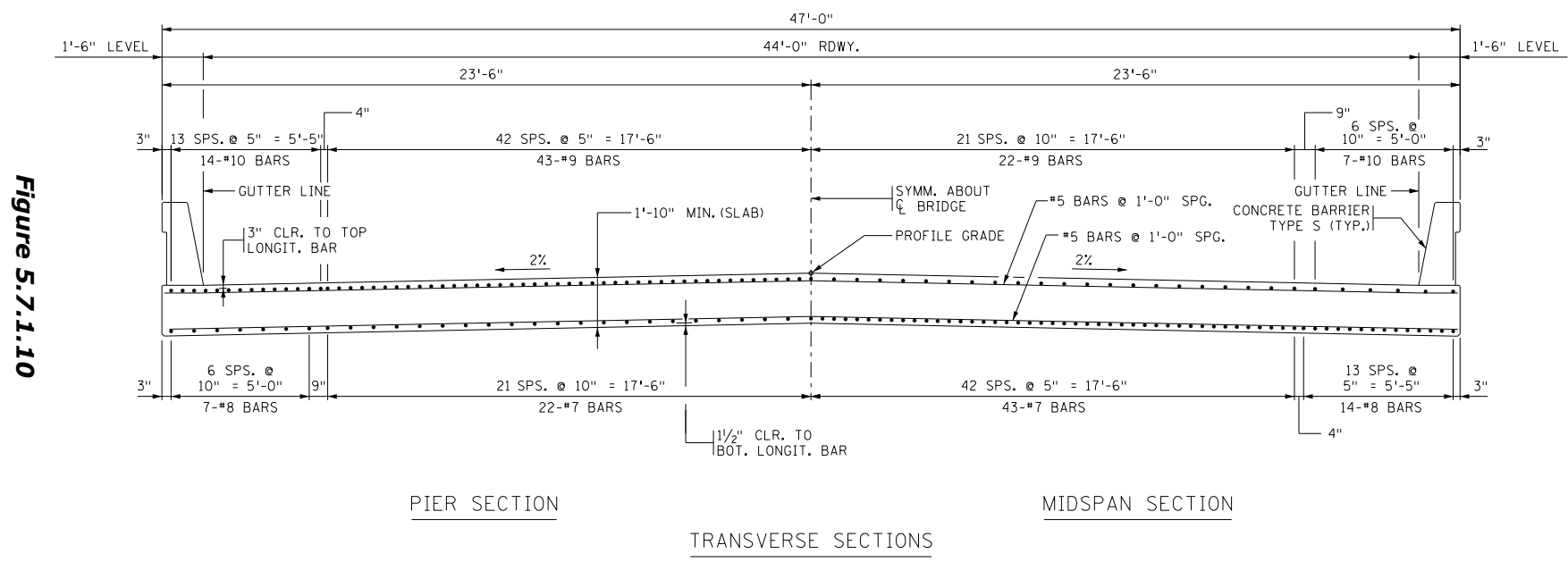


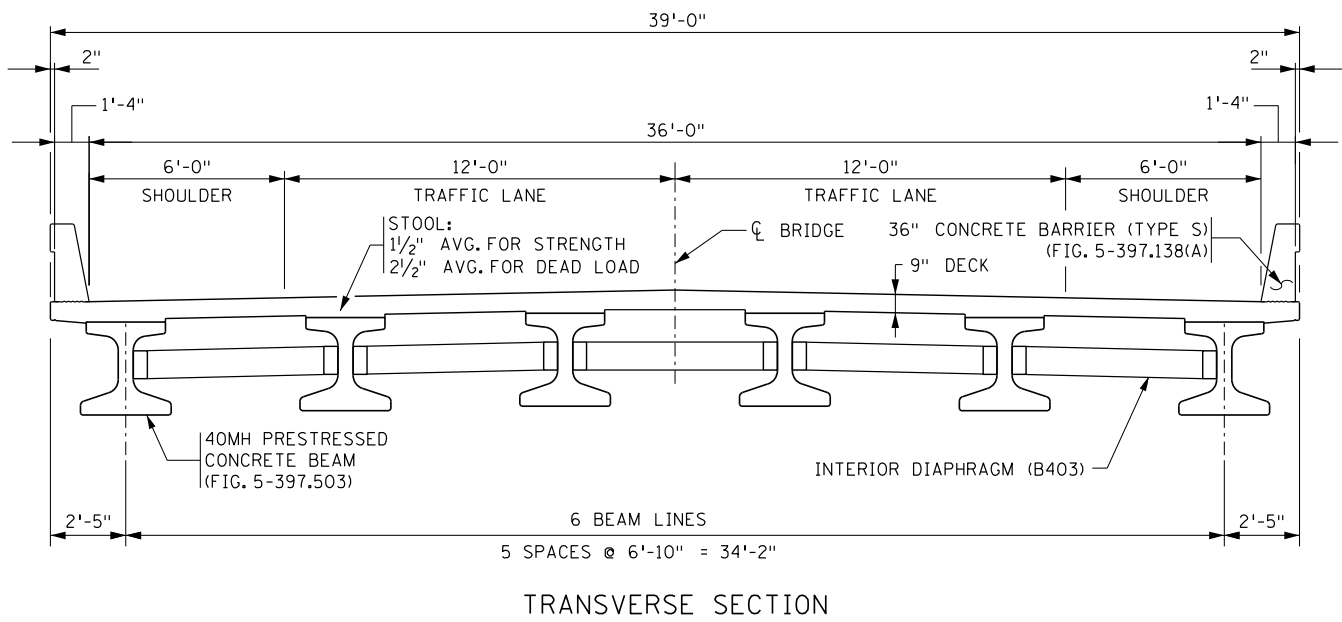
Figure 5.7.1.10

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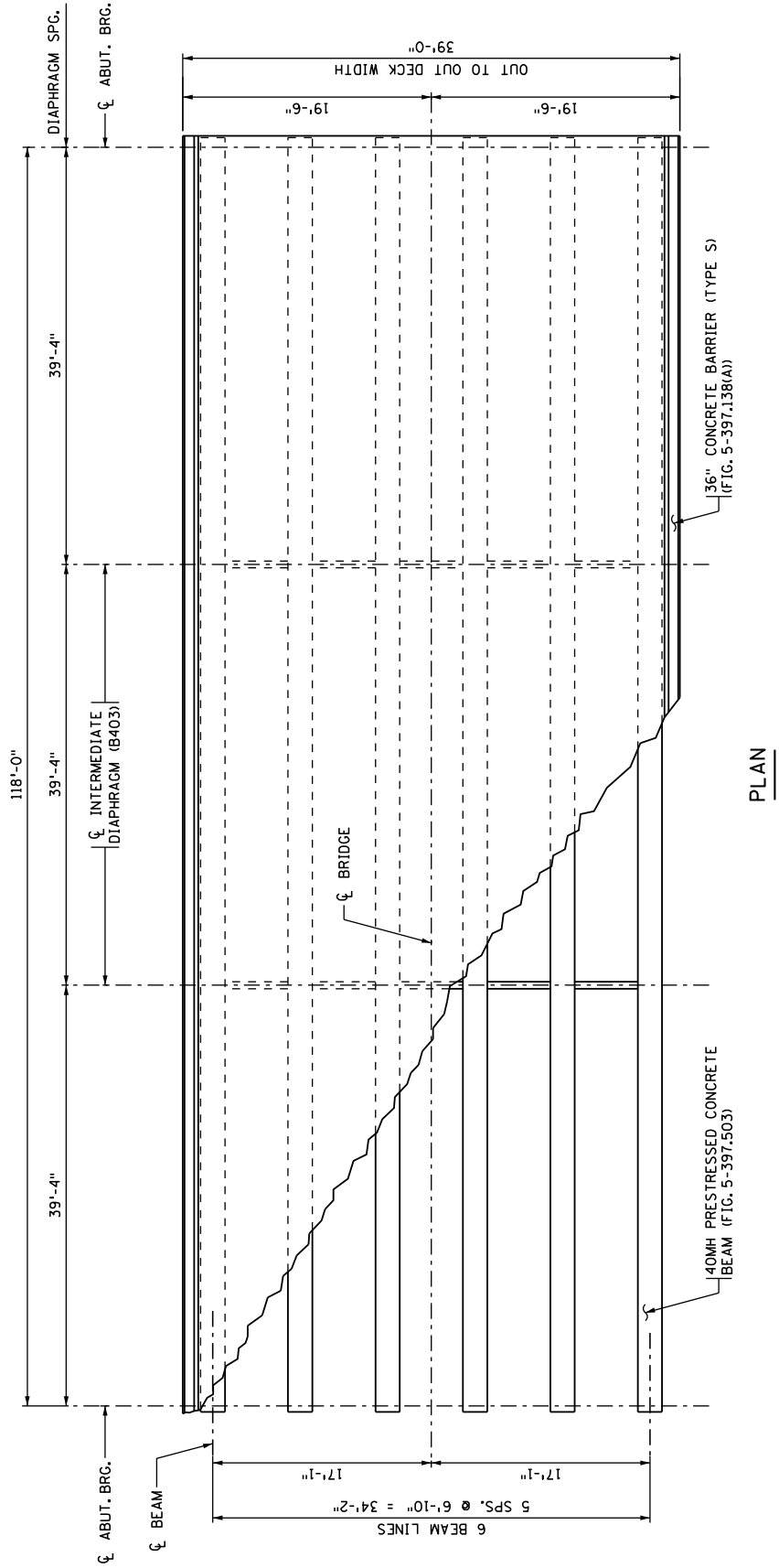
**5.7.2 Draped  
Prestressed I-  
Beam Design  
Example**

This example illustrates the design of a pretensioned I-beam for a single span bridge without skew. The 118'-0" span is supported with MnDOT "40MH" beams on integral abutments. MnDOT standard details and drawings for diaphragms (B403), barriers (Fig. 5-397.138(A)), and beams (Fig. 5-397.503) are to be used with this example. This example contains the design of a typical interior beam at the critical sections in positive flexure, shear, and deflection. The superstructure consists of six beams spaced at 6'-10" centers. A typical transverse superstructure section is provided in Figure 5.7.2.1. A framing plan is provided in Figure 5.7.2.2. The roadway section is composed of two 12' traffic lanes and two 6' shoulders. A Type S barrier is provided on each side of the bridge and a 9" monolithic concrete deck is used. Interior diaphragms are used at the interior third points based on guidance found in BDM Table 5.4.1.1.

This example uses 0.6" diameter, 300 ksi, low relaxation strands for pretensioning. Draped strands are used to control the beam end stresses.



**Figure 5.7.2.1**



PLAN



**A. Materials**

The modulus of elasticity  $E_c$  for high strength concrete suggested by ACI Committee 363 is used for the beam concrete. AASHTO Article 5.4.2.4 is used to calculate  $E_c$  for the deck and assumes a  $K_1$  equal to 1.0. The composite deck is assumed to have a unit weight of 0.150 kcf for dead load computations and 0.145 kcf for  $E_c$  computations. The beam concrete is assumed to have a unit weight of 0.155 kcf for dead load computations.

The material and geometric parameters used in the example are shown in Table 5.7.2.1:

**Table 5.7.2.1  
Material Properties**

Material Parameter		Prestressed Beam	Deck
Concrete	$f'_{ci}$ at transfer	8.0 ksi *	---
	$f'_c$ at 28 days	9.5 ksi *	4 ksi
	$E_{ci}$ at transfer	$(1265 \cdot \sqrt{f'_{ci}}) + 1000$ = 4578 ksi	---
	$E_c$ at 28 days	$(1265 \cdot \sqrt{f'_c}) + 1000$ = 4899 ksi	$120,000 \cdot K_1 \cdot (w_c)^2 \cdot (f'_c)^{0.33}$ = 3987 ksi
Steel	$f_y$ for rebar	60 ksi	60 ksi
	$f_{pu}$ for strand	300 ksi	---
	$E_s$ for rebar	29,000 ksi	29,000 ksi
	$E_p$ for strand	28,500 ksi	---
	Strand type	0.6 inch diameter 300 ksi, low relaxation	---

\*These concrete compressive strength values are initial assumed values. Final values may differ based on adjustments for the actual and initial final service stresses.

**B. Determine Cross-Section Properties for a Typical Interior Beam**

The beams are designed to act compositely with the deck on simple spans. The deck consists of a 9 inch thick concrete slab. A 1/2 inch of wear is assumed. A thickness of 8 1/2 inches is used for composite section properties. The stool height is assumed to be an average of 2 1/2 inches for dead load computations and 1 1/2 inches for section property computations.

**[4.6.2.6.1]**

The effective flange width,  $b_e$ , is equal to the average beam spacing:

$$b_e = 82.00 \text{ in}$$

To transform the deck and stool concrete to beam concrete, use a modular ratio  $n_{d\_bm}$  based on  $E_{cdeck}$  to  $E_{cbeam}$ :

$$n_{d\_bm} = \frac{E_{cdeck}}{E_{cbeam}} = \frac{3987}{4899} = 0.81$$

This results in a transformed effective flange width of:

$$b_{\text{trans}} = n_{d\_bm} \cdot b_e = 0.81 \cdot 82 = 66.42 \text{ in}$$

Properties for an interior beam are given in Table 5.7.2.2.

**Table 5.7.2.2  
Cross-Section Properties**

Parameter	Non-composite Section	Composite Section
Height of section, h	40.00 in	50.00 in
Deck thickness	---	8.50 in
Average stool thickness	---	1.50 in (section properties) 2.50 in (dead load)
Effective flange width, $b_e$	---	82.00 in (deck concrete) 66.42 in (beam concrete)
Area, A	704 in <sup>2</sup>	1310 in <sup>2</sup>
Moment of inertia, I	149,002 in <sup>4</sup>	396,823 in <sup>4</sup>
Centroidal axis height, y	18.07 in	30.72 in
Bottom section modulus, $S_b$	8246 in <sup>3</sup>	12,917 in <sup>3</sup>
Top section modulus, $S_t$	6794 in <sup>3</sup>	25,410 in <sup>3</sup>
Top of prestressed beam, $S_{t_{bm}}$	6794 in <sup>3</sup>	42,761 in <sup>3</sup>

**C. Live Load  
Distribution Factors  
and Load Modifiers**

Assume that traffic can be positioned anywhere between the barriers.

$$\text{Number of design lanes} = \frac{\text{distance between barriers}}{\text{design lane width}} = \frac{36}{12} = 3$$

**[4.6.2.2]**

**1. Determine Live Load Distribution Factors**

Designers should note that the approximate live load distribution factor equations include the multiple presence factors.

**[4.6.2.2.2]**

**Live Load Distribution Factor for Moment – Interior Beams**

LRFD Table 4.6.2.2.1-1 lists the common deck superstructure types for which approximate live load distribution equations have been assembled. The cross section for this design example is Type (k). To ensure that the approximate distribution equations can be used, several parameters need to be checked.

- 1) 3.5 ft ≤ beam spacing = 6.83 ft ≤ 16.0 ft    OK
- 2) 4.5 in ≤ slab thickness = 8.5 in ≤ 12.0 in    OK
- 3) 20 ft ≤ span length = 118 ft ≤ 240 ft    OK
- 4) 4 ≤ number of beams = 6    OK

The live load distribution factor equations use a  $K_g$  factor that is defined in LRFD Article 4.6.2.2.1. For determination of  $K_g$ , the beam concrete is transformed to deck concrete, so the modular ratio  $n_{bm\_d}$  differs from  $n_{d\_bm}$  calculated earlier.

$$n_{bm\_d} = \frac{E_{cbeam}}{E_{cdeck}} = \frac{4899}{3987} = 1.23$$

$$e_g = (\text{deck centroid}) - (\text{beam centroid}) = 45.75 - 18.07 = 27.68 \text{ in}$$

$$K_g = n_{bm\_d} \cdot [I + A \cdot (e_g)^2] = 1.23 \cdot [149,002 + 704 \cdot (27.68)^2] \\ = 8.47 \times 10^5 \text{ in}^4$$

Check  $K_g$  limits:  $1 \times 10^4 \leq K_g = 8.47 \times 10^5 \leq 7 \times 10^6$  OK

For one design lane loaded, the live load distribution factor for moment,  $gM_{int\_1lane}$ , is:

$$gM_{int\_1lane} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12 \cdot L \cdot t_s^3}\right)^{0.1}$$

$$gM_{int\_1lane} = 0.06 + \left(\frac{6.83}{14}\right)^{0.4} \cdot \left(\frac{6.83}{118}\right)^{0.3} \cdot \left(\frac{8.47 \times 10^5}{12 \cdot 118 \cdot 8.5^3}\right)^{0.1}$$

$$gM_{int\_1lane} = 0.378 \text{ lanes/beam}$$

Two or more design lanes loaded:

$$gM_{int\_mlane} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K_g}{12 \cdot L \cdot t_s^3}\right)^{0.1}$$

$$gM_{int\_mlane} = 0.075 + \left(\frac{6.83}{9.5}\right)^{0.6} \cdot \left(\frac{6.83}{118}\right)^{0.2} \cdot \left(\frac{8.47 \times 10^5}{12 \cdot 118 \cdot 8.5^3}\right)^{0.1}$$

$$gM_{int\_mlane} = 0.538 \text{ lanes/beam}$$

**[4.6.2.2.2d]**

**Live Load Distribution Factor for Moment - Exterior Beams**

LRFD Table 4.6.2.2.2d-1 contains the approximate live load distribution factor equations for exterior beams. Type (k) cross-sections have a deck dimension check to ensure that the approximate equations are valid. The distance from the inside face of barrier to the centerline of the fascia beam is defined as  $d_e$ . For the example this distance is:

$$d_e = \text{deck overhang} - \text{deck coping} - \text{barrier width} \\ = \frac{(29 - 2 - 16)}{12} = 0.92 \text{ ft}$$

Check whether approximate equations can be used:

$$-1.0 \text{ ft} \leq d_e = 0.92 \text{ ft} \leq 5.5 \text{ ft} \quad \text{OK}$$

One design lane loaded:

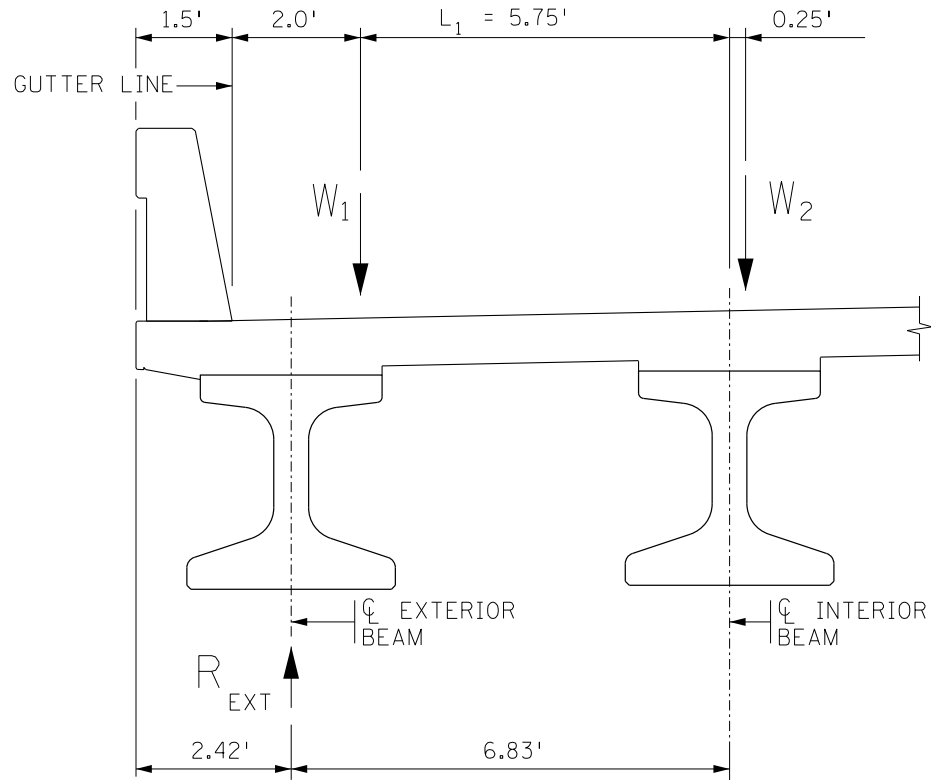


Figure 5.7.2.3

Use the lever rule to determine the live load distribution factor for one lane. The exterior beam live load distribution factor is found by determining the exterior beam reaction and applying the multiple presence factor,  $m$ , for one lane:

[Table 3.6.1.1.2-1]

$$W_1 = W_2 = 0.5 \text{ lanes}$$

$$gM_{\text{ext}_1\text{lane}} = R_{\text{ext}} \cdot m = \left( \frac{W_1 \cdot L_1}{S} \right) \cdot m = \left( \frac{0.5 \cdot 5.75}{6.83} \right) \cdot 1.20$$

$$gM_{\text{ext}_1\text{lane}} = 0.505 \text{ lanes/beam}$$

Two or more design lanes loaded:

The live load distribution factor is equal to the factor “e” multiplied by the interior girder live load distribution factor for two or more lanes.

$$e = 0.77 + \left(\frac{d_e}{9.1}\right) = 0.77 + \left(\frac{0.92}{9.1}\right) = 0.871$$

$$gM_{\text{ext\_mlane}} = e \cdot gM_{\text{int\_mlane}} = 0.871 \cdot 0.538 = 0.469 \text{ lanes/beam}$$

**[4.6.2.2.2e]****Skew Factor**

No correction is necessary for a skew angle of zero.

**[4.6.2.2.3]****[4.6.2.2.3a]****Live Load Distribution Factor for Shear – Interior Beams**

LRFD Table 4.6.2.2.3a-1 can be used.

One design lane loaded:

$$gV_{\text{int\_1lane}} = 0.36 + \left(\frac{S}{25.0}\right) = 0.36 + \left(\frac{6.83}{25}\right) = 0.633 \text{ lanes/beam}$$

Two or more design lanes loaded:

$$gV_{\text{int\_mlane}} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^2 = 0.2 + \left(\frac{6.83}{12}\right) - \left(\frac{6.83}{35}\right)^2$$

$$= 0.731 \text{ lanes/beam}$$

**[4.6.2.2.3b]****Live Load Distribution Factor for Shear – Exterior Beams**

One Design Lane Loaded:

Use the lever rule, which results in the same factor that was computed for flexure, which is 0.505 lanes/beam.

Two or more design lanes loaded:

$$e = 0.6 + \left(\frac{d_e}{10}\right) = 0.6 + \left(\frac{0.92}{10}\right) = 0.692$$

The exterior beam shear live load distribution factor for two or more design lanes is determined by modifying the interior distribution factor:

$$gV_{\text{ext\_mlane}} = e \cdot gV_{\text{int\_mlane}} = 0.692 \cdot 0.731 = 0.506 \text{ lanes/beam}$$

**[4.6.2.2.3c]****Skew Factor**

No correction is necessary for a skew angle of zero.

**[2.5.2.6.2]****[Table 3.6.1.1.2-1]****Live Load Distribution Factor for Deflection**

The live load distribution factor for checking live load deflections assumes that the entire cross section participates in resisting the live load. The live load distribution factor for deflection is:

$$gD = \frac{(\# \text{ of lanes}) \cdot m}{(\# \text{ of beam lines})} = \frac{3 \cdot 0.85}{6} = 0.425 \text{ lanes/beam}$$

**Live Load Distribution Factor for Fatigue – Interior and Exterior Beams**

[3.6.1.1.2]

The fatigue limit state is to be analyzed for one traffic lane, but the multi-presence factor does not apply. The live load distribution factor for one lane is to be divided by 1.2 to account for this.

Interior Beam:

$$g_{F_{int\_1lane}} = \frac{gM_{int\_1lane}}{1.2} = \frac{0.378}{1.2} = 0.315 \text{ lanes/beam}$$

Exterior Beam:

$$g_{F_{ext\_1lane}} = \frac{gM_{ext\_1lane}}{1.2} = \frac{0.505}{1.2} = 0.421 \text{ lanes/beam}$$

Table 5.7.2.3 contains a summary of the live load distribution factors and Table 5.7.2.4 contains a summary of the load modifiers for this example.

**Table 5.7.2.3  
Live Load Distribution Factor Summary (lanes per beam)**

Loading		Flexure	Shear	Deflection	Fatigue
Interior Beam	One Design Lane	0.378	0.633	-	0.315
	Two or More Design Lanes	0.538	0.731	0.425	-
Exterior Beam	One Design Lane	0.505	0.505	-	0.421
	Two or More Design Lanes	0.469	0.506	0.425	-

[1.3.3 – 1.3.5]

**Table 5.7.2.4 Load Modifiers**

Modifier	Strength	Service	Fatigue
Ductility $\eta_D$	1.0	1.0	1.0
Redundancy $\eta_R$	1.0	1.0	1.0
Importance $\eta_I$	1.0	n/a	n/a
$\eta = \eta_D \cdot \eta_R \cdot \eta_I$	1.0	1.0	1.0

**D. Shear Forces  
and Bending  
Moments**

Four load combinations will be considered; Strength I, Service I, Service III, and Fatigue. As a result of the simple span configuration, only maximum  $\gamma_p$  values need to be considered.

Load effects related to settlement, thermal effects, water load, or stream pressure will not be considered.

**[3.6.2]**

Dynamic load allowance IM = 33%

$$\text{Beam Selfweight} = (704/144) \cdot (0.155 \text{ k/ft}^3) = 0.758 \text{ k/ft}$$

$$\text{Stool Weight} = (2.83 \text{ ft}) \cdot (0.208 \text{ ft}) \cdot (0.150 \text{ k/ft}^3) = 0.088 \text{ k/ft}$$

$$\text{Deck Weight} = (6.83 \text{ ft}) \cdot (0.75 \text{ ft}) \cdot (0.150 \text{ k/ft}^3) = 0.769 \text{ k/ft}$$

$$\text{Future Wearing Surface} = (0.020 \text{ k/ft}^2) \cdot (36 \text{ ft}) \cdot (1/6) = 0.120 \text{ k/ft}$$

$$\text{Barrier Weight} = 2 \cdot (0.496 \text{ k/ft}) \cdot (1/6) = 0.165 \text{ k/ft}$$

The load due to the intermediate diaphragms is calculated by referring to standard detail B403. For 40MH beams, the diaphragm consists of a steel C12 x 20.7 that is connected to the beams with 1.0' x 1.0' bent plates.

$$\text{Diaphragm Weight} \cong (6.83 \text{ ft}) \cdot (0.0207 \text{ k/ft})$$

$$+ 2 \cdot (1.0 \text{ ft}) \cdot (1.0 \text{ ft}) \cdot \left( \frac{0.375 \text{ in}}{12 \text{ in/ft}} \right) \cdot (0.490 \text{ k/ft}^3) = 0.172 \text{ kips}$$

Critical locations along the beam need to be analyzed for moments and shear. These critical locations include: the inside face of bearing, prestress transfer point, critical shear point, and tenth points along the length of the beam. These locations, dimensioned from the beam centerline of bearing, are determined as follows:

Bearing Face (inside face of bearing is the point where a crack could start at the bottom of the beam, which is the inside edge of the sole plate)

$$= X_{\text{brgface}} = \frac{L_{\text{soleplate}}}{2} = 7.5 \text{ in} = 0.63 \text{ ft}$$

Transfer Point

$$= X_{\text{transfer}} = 60 \cdot d_b - \frac{L_{\text{soleplate}}}{2} = (60 \cdot 0.6) - \frac{15}{2} = 28.5 \text{ in} = 2.38 \text{ ft}$$

Critical Shear Point (located at  $d_v$  from the inside face of bearing, calculations are shown in "F. Design Reinforcement for Shear")

$$= X_{\text{vcrit}} = 4.07 \text{ ft}$$

Tenth points are simply 0.1L, 0.2L, 0.3L, 0.4L, and 0.5L.

The bending moments and shears for the dead and live loads were obtained with a line girder model of the bridge. They are summarized in Tables 5.7.2.4 and 5.7.2.5.

**Table 5.7.2.4**  
**Shear Force Summary (kips/beam)**

Load Type/Combination		Brg CL (0.0')	Brg Face (0.63')	Trans Point (2.38')	Critical Shear Point (4.1')	0.1 Span Point (11.8')	0.2 Span Point (23.6')	0.3 Span Point (35.4')	0.4 Span Point <sup>②</sup> (47.2')	0.5 Span Point (59.0')
Dead Loads	Selfweight	45	44	43	42	36	27	18	9	0
	Stool	5	5	5	5	4	3	2	1	0
	Deck	45	45	44	42	36	27	18	9	0
	FWS	7	7	7	7	6	4	3	1	0
	Barrier	10	10	9	9	8	6	4	2	0
	Diaphragms	0	0	0	0	0	0	0	0	0
	Total	112	111	108	105	90	67	45	22	0
Live Loads <sup>①</sup>	Uniform Lane	28	27	27	26	22	18	14	10	7
	Tandem + IM	48	48	47	46	43	38	33	28	23
	Truck + IM	64	64	63	62	57	50	43	36	29
	Governing LL Total (Truck + IM) + Lane	92	91	90	88	79	68	57	46	36
Strength I Load Comb (1.25 · DL + 1.75 · LL)		301	298	293	285	251	203	156	108	63
Service I Load Comb (1.00 · DL + 1.00 · LL)		204	202	198	193	169	135	102	68	36
Service III Load Comb (1.00 · DL + 0.80 · LL)		186	184	180	175	153	121	91	59	29

① All live loads include the interior beam live load distribution factor of 0.731 and IM of 0.33.

② Hold down point for draped strands.



**Table 5.7.2.5**  
**Bending Moment Summary (kip-ft/beam)**

Load Type/Combination		Brg CL (0.0')	Brg Face (0.63')	Trans Point (2.38')	Critical Shear Point (4.1')	0.1 Span Point (11.8')	0.2 Span Point (23.6')	0.3 Span Point (35.4')	0.4 Span Point (47.2') ②	0.5 Span Point (59.0')	
Dead Loads	DC1	Selfweight	0	28	104③	177	475	844	1108	1267④	1319
		Stool	0	3	12	21	55	98	129	147	153
		Deck	0	28	106	180	482	857	1124	1285	1338
		Diaphragms	0	0	0	1	2	4	6	7	7
		Total DC1	0	59	222	379	1014	1803	2367	2706	2817
	DC2	Barrier	0	6	23	39	103	184	241	276	287
		FWS	0	4	16	28	75	134	175	201	209
		Total DC2	0	10	39	67	178	318	416	477	496
	Total (DC1+DC2)		0	69	261	446	1192	2121	2783	3183	3313
Live Loads ①	Uniform Lane	0	13	47	80	216	384	503	575	599	
	Tandem + IM	0	22	82	139	373	661	865	985	1020	
	Truck + IM	0	29	110	187	499	877	1132	1283	1319	
	Governing LL Total (Truck + IM) + Lane	0	42	157	267	715	1261	1635	1858	1918⑤	
Strength I - Load Comb (1.25 · DL + 1.75 · LL)		0	160	601	1025	2741	4858	6340	7230	7498	
Service I - Load Comb (1.00 · DL + 1.00 · LL)		0	111	418	713	1907	3382	4418	5041	5231	
Service III - Load Comb (1.00 · DL + 0.80 · LL)		0	103	387	660	1764	3130	4091	4669	4847	
Fatigue I - Load Comb (1.75 · LL)		-	-	-	-	-	-	-	-	1008	

① All live loads include the interior beam live load distribution factor of 0.538 and IM of 0.33.

② Hold down point for draped strands.

③ Beam selfweight at strand release = 132 k-ft (beam in casting bed with span length equal to overall beam length of 119.25 ft).

Beam selfweight at erection on bearings = 104 k-ft (beam span length equal to design span of 118.0 ft).

④ Beam selfweight at strand release = 1295 k-ft (beam in casting bed with span length equal to overall beam length of 119.25 ft).

Beam selfweight at erection on bearings = 1267 k-ft (beam span length equal to design span of 118.0 ft).

⑤ Fatigue live load = 576 k-ft (includes interior beam live load distribution factor of 0.315 and IM of 0.15 applied to fatigue truck only).

**E. Design Beam  
Pretensioning With  
Draped Strands for  
Control of End  
Stresses**

Typically the tension at the bottom of the beam at midspan in its final configuration after all losses have occurred dictates the required level of prestressing.

**1. Estimate Required Prestress**

Use the Service III load combination

Bottom of beam stress:

$$f_{\text{serv3bot}} = \left( \frac{M_{\text{DC1}}}{S_{\text{gb}}} \right) + \left( \frac{M_{\text{DC2}}}{S_{\text{cb}}} \right) + \left( \frac{M_{\text{LL}} \cdot 0.8}{S_{\text{cb}}} \right)$$

$$= \left( \frac{2817 \cdot 12}{8246} \right) + \left( \frac{496 \cdot 12}{12,917} \right) + \left( \frac{1918 \cdot 12 \cdot 0.8}{12,917} \right) = 5.99 \text{ ksi}$$

For 300 ksi strands, MnDOT practice is to jack to an initial prestress force of  $0.72f_{\text{pu}}$ . As a starting point, the total prestress losses will be assumed to be 25%. This results in an effective prestress of

$$f_{\text{pe}} = 0.72 \cdot f_{\text{pu}} \cdot (1 - 0.25) = 0.72 \cdot 300 \cdot 0.75 = 162.0 \text{ ksi}$$

Strands are typically placed on a 2" grid. Referring to BDM Figures 5.4.6.2 to determine a starting point for the number of strands, assume 52 strands and choose a pattern that provides the greatest eccentricity for the prestressing force. This pattern will fill all the straight strand locations in the bottom flange and include 8 draped strands. The centroid of this 52 strand pattern is:

$$y_{\text{str}} = \left[ \frac{\sum (\# \text{ of strands}) \cdot (y \text{ of strands})}{(\text{total } \# \text{ of strands})} \right]$$

$$= \left[ \frac{18 \cdot (2 + 4) + (10 \cdot 6) + (4 \cdot 8) + 2 \cdot (10)}{52} \right] = 4.23 \text{ in}$$

Using the centroid of this group as an estimate of the strand pattern eccentricity results in

$$e_{52} = y_{\text{g}} - y_{\text{str}} = 18.07 - 4.23 = 13.84$$

The area,  $A_{\text{strand}}$ , of a 0.6" diameter 7-wire strand is  $0.217 \text{ in}^2$ .

The axial compression produced by the prestressing strands is

$$P = A_s \cdot f_{pe} = n_{\text{strands}} \cdot A_{\text{strand}} \cdot f_{pe}$$

The internal moment produced by the prestressing strands is

$$M_{p/s} = A_s \cdot f_{pe} \cdot e_{52} = n_{\text{strands}} \cdot A_{\text{strand}} \cdot f_{pe} \cdot e_{52}$$

The allowable tension after losses =  $0.19 \cdot \sqrt{f'_c} = 0.19 \cdot \sqrt{9.5} = 0.59$  ksi

This moment and axial compression from the prestress,  $f_{pscomp}$ , must reduce the bottom flange tension from 5.99 ksi tension to the allowable tension of 0.59 ksi.

$$f_{pscomp} = 5.99 - 0.59 = 5.40 \text{ ksi}$$

Knowing that

$$f_{pscomp} = \frac{P}{A} + \frac{M_{p/s}}{S_b}$$

and substituting and solving for  $n_{\text{strands}}$ , we get an estimate for the required number of strands:

$$\begin{aligned} n_{\text{strands}} &= \frac{f_{pscomp}}{A_{\text{strand}} \cdot f_{pe} \cdot \left( \frac{1}{A} + \frac{e_{52}}{S_b} \right)} = \frac{5.40}{0.217 \cdot 162.0 \cdot \left( \frac{1}{704} + \frac{13.84}{8246} \right)} \\ &= 49.6 \text{ strands} \end{aligned}$$

Try a strand pattern with 50 strands.

After reviewing Bridge Details Part II Figure 5-397.503, a trial strand pattern with maximum prestress eccentricity and 50 strands was selected, but calculations showed it did not meet the Service I compression stress limit at the beam ends immediately after strand release. A second trial strand pattern (see Figure 5.7.2.4) was then selected. The drape points were chosen to be at  $0.40L = 47.2$  ft from the centerline of bearing locations.

The properties of this strand pattern at midspan are:

$$y_{\text{strand}} = \left[ \frac{(18 \cdot 2) + (16 \cdot 4) + (10 \cdot 6) + (4 \cdot 8) + (2 \cdot 10)}{50} \right] = 4.24 \text{ in}$$

$$e_{\text{strand}} = y_b - y_{\text{strand}} = 18.07 - 4.24 = 13.83 \text{ in}$$

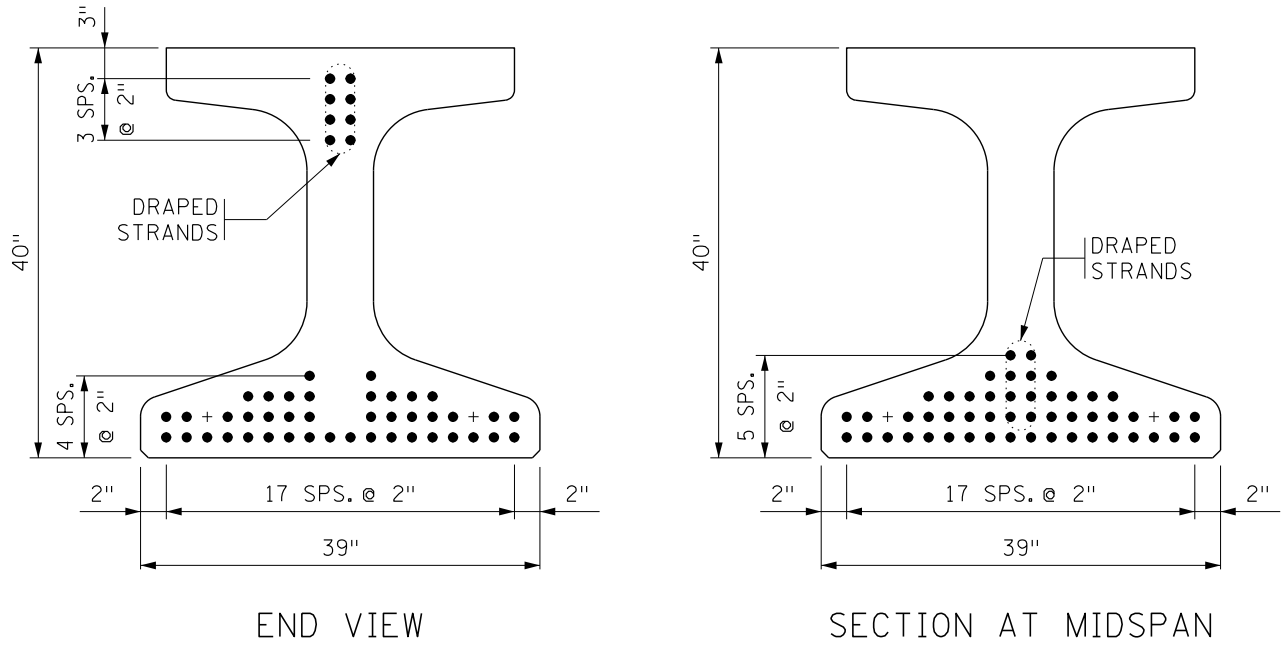


Figure 5.7.2.4

[5.9.3]

**2. Prestress Losses**

Prestress losses are computed using the approximate method.

[5.9.3.2.3]

**Elastic Shortening Loss**

Use the alternative equation presented in the LRFD C5.9.3.2.3a:

$$\Delta f_{pES} = \frac{A_{ps} \cdot f_{pbt} \cdot (I_g + e_m^2 \cdot A_g) - e_m \cdot M_g \cdot A_g}{A_{ps} \cdot (I_g + e_m^2 \cdot A_g) + \frac{A_g \cdot I_g \cdot E_{ci}}{E_p}}$$

$$A_{ps} = (\# \text{ of strands}) \cdot (\text{strand area}) = 50 \cdot 0.217 = 10.85 \text{ in}^2$$

$$f_{pbt} = f_{pj} = 216.0 \text{ ksi}$$

$$e_m = e_{strand} = 13.83 \text{ in}$$

$$\frac{A_g \cdot I_g \cdot E_{ci}}{E_p} = \frac{704 \cdot 149,002 \cdot 4578}{28,500} = 16,849,836 \text{ in}^6$$

$$A_{ps} \cdot (I_g + e_m^2 \cdot A_g) = 10.85 [149,002 + (13.83)^2 \cdot (704)] = 3,077,660 \text{ in}^6$$

$$\Delta f_{pES} = \frac{216.0 \cdot (3,077,660) - 13.83 \cdot (1319) \cdot (12) \cdot (704)}{3,077,660 + 16,849,836} = 25.6 \text{ ksi}$$

[5.9.3.3]

**Long Term Losses**

Use the approximate equation in the LRFD 5.9.3.3

$$\Delta f_{pLT} = 10.0 \cdot \frac{f_{pi} \cdot A_{ps}}{A_g} \cdot \gamma_h \cdot \gamma_{st} + 12 \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR}$$

For an average humidity in Minnesota of 73%

$$\gamma_h = 1.7 - 0.01 \cdot H = 1.7 - 0.01 \cdot 73 = 0.97$$

$$\gamma_{st} = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 8.0} = 0.56$$

For low relaxation strand,  $\Delta f_{pR} = 2.4$

$$\begin{aligned} \Delta f_{pLT} &= 10.0 \cdot \frac{216.0 \cdot (10.85)}{704} \cdot 0.97(0.56) + 12.0(0.97)(0.56) + 2.4 \\ &= 27.0 \text{ ksi} \end{aligned}$$

### [5.9.3.1]

#### Total Losses

$$\Delta f_{pt} = \Delta f_{pES} + \Delta f_{pLT} = 25.6 + 27.0 = 52.6 \text{ ksi}$$

$$f_{pe} = f_{pj} - \Delta f_{pt} = 216.0 - 52.6 = 163.4 \text{ ksi}$$

$$\text{prestress loss percentage} = \frac{\Delta f_{pt}}{f_{pj}} \cdot 100 = \frac{52.6}{216.0} \cdot 100 = 24.4 \%$$

Jacking force:

$$P_{jack} = A_{ps} \cdot (f_{pj}) = 10.85 \cdot (216.0) = 2344 \text{ kips}$$

Initial prestress force after transfer:

$$P_i = A_{ps} \cdot (f_{pj} - \Delta f_{pES}) = 10.85 \cdot (216.0 - 25.6) = 2066 \text{ kips}$$

Prestress force after all losses:

$$P_e = A_{ps} \cdot f_{pe} = 10.85 \cdot 163.4 = 1773 \text{ kips}$$

### [5.9.2.3.1]

#### 3. Stresses at Transfer (compression +, tension -) Stress Limits for P/S Concrete at Release

Compression in the concrete is limited to:

$$f_{climrel} = 0.65 \cdot f'_{ci} = 0.65 \cdot 8.0 = 5.20 \text{ ksi}$$

For tension, MnDOT uses the AASHTO Table 5.9.2.3.1b-1 stress limits for "areas other than the precompressed tensile zone without bonded reinforcement" for beams designed with draped strands. The limit is the smaller tension stress of:

$$f_{tlimrel1} = -0.0948 \cdot \sqrt{f'_{ci}} = -0.0948 \cdot \sqrt{8.0} = -0.268 \text{ ksi}$$

or

$$f_{tlimrel2} = -0.200 \text{ ksi}$$

$$\text{Then } f_{tlimrel} = -0.200 \text{ ksi}$$

**Check Release Stresses at Drape Point (0.40 Point of Span)**

At this point, the beam is sitting in the casting bed. The beam will camber upward when the strands are released, so the span length used to determine the selfweight moment is the end-to-end beam length of 119.25 feet.

$$P_i \cdot e_{\text{strand}} = 2066 \cdot 13.83 = 28,573 \text{ kip-in}$$

$$\begin{aligned} \text{Top stress due to P/S} &= \left( \frac{P_i}{A_g} \right) - \left( \frac{P_i \cdot e_{\text{strand}}}{S_{gt}} \right) = \left( \frac{2066}{704} \right) - \left( \frac{28,573}{6794} \right) \\ &= -1.27 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{Bottom stress due to P/S} &= \left( \frac{P_i}{A_g} \right) + \left( \frac{P_i \cdot e_{\text{strand}}}{S_{gb}} \right) = \left( \frac{2066}{704} \right) + \left( \frac{28,573}{8246} \right) \\ &= 6.40 \text{ ksi} \end{aligned}$$

$$\text{Selfweight moment at drape point} = M_{\text{sw}0.40} = 1295 \text{ kip-ft}$$

$$\text{Top stress due to selfweight} = \left( \frac{M_{\text{sw}0.40}}{S_{gt}} \right) = \left( \frac{1295 \cdot 12}{6794} \right) = 2.29 \text{ ksi}$$

$$\text{Bottom stress due to selfweight} = \left( \frac{M_{\text{sw}0.40}}{S_{gb}} \right) = \left( \frac{1295 \cdot 12}{8246} \right) = -1.88 \text{ ksi}$$

$$\text{Top stress at drape point} = -1.27 + 2.29 = 1.02 \text{ ksi}$$

$$f_{\text{timrel}} = -0.200 \text{ ksi} \quad \text{OK}$$

$$\text{Bottom stress at drape point} = 6.40 - 1.88 = 4.52 \text{ ksi} < 5.20 \text{ ksi} \quad \text{OK}$$

**Check Release Stresses at End of Beam**

The strands need to be draped to raise the eccentricity of the prestress force and limit the potential for cracking the top of the beams. Stresses are checked at the transfer point (60 bar diameters from the end of the beam), using the total length of the beam for selfweight moment calculations.

Centroid of strand pattern at the end of the beams:

$$\begin{aligned} y_{\text{strand}} &= \left[ \frac{(18 \cdot 2) + (14 \cdot 4) + (8 \cdot 6) + 2 \cdot (37+35+33+31+8)}{50} \right] \\ &= 8.56 \text{ in} \end{aligned}$$

Centroid of strand at the transfer point:

$$y_{\text{strand}} = 8.56 - \frac{60 \cdot 0.6}{118 \cdot 0.4 \cdot 12 + 7.5} \cdot (8.56 - 4.24) = 8.29 \text{ in}$$

The eccentricity of the strand pattern at the transfer point is:

$$e_{\text{strand}} = y_b - y_{\text{strand}} = 18.07 - 8.29 = 9.78 \text{ in}$$

The internal prestress moment is:

$$P_i \cdot e_{\text{strand}} = 2066 \cdot 9.78 = 20,205 \text{ kip-in}$$

$$\begin{aligned} \text{Top stress due to } P/S &= \left( \frac{P_i}{A_g} \right) - \left( \frac{P_i \cdot e_{\text{strand}}}{S_{gt}} \right) = \left( \frac{2066}{704} \right) - \left( \frac{20,205}{6794} \right) \\ &= -0.039 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{Bottom stress due to } P/S &= \left( \frac{P_i}{A_g} \right) + \left( \frac{P_i \cdot e_{\text{strand}}}{S_{gb}} \right) = \left( \frac{2066}{704} \right) + \left( \frac{20,205}{8246} \right) \\ &= 5.38 \text{ ksi} \end{aligned}$$

$$\text{Top stress due to selfweight} = \left( \frac{M_{\text{swtr}}}{S_{gt}} \right) = \left( \frac{132 \cdot 12}{6794} \right) = 0.233 \text{ ksi}$$

$$\text{Bottom stress due to selfweight} = - \left( \frac{M_{\text{swtr}}}{S_{gb}} \right) = - \left( \frac{132 \cdot 12}{8246} \right) = -0.192 \text{ ksi}$$

$$\text{Top stress at transfer point} = -0.039 + 0.233 = 0.194 \text{ ksi}$$

$$f_{\text{tlimrel}} = -0.200 \text{ ksi} \quad \text{OK}$$

$$\text{Bottom stress at transfer point} = 5.38 - 0.192 = 5.19 \text{ ksi} < 5.20 \text{ ksi} \quad \text{OK}$$

The initial concrete strength,  $f'_{ci}$ , was assumed to be 8.0 ksi. For the most economical beam, the designer should choose the lowest required  $f'_{ci}$  for the beam. This is determined by substituting the calculated maximum compression stress for  $f_{\text{climrel}}$  in the compression limit equation and solving for  $f'_{ci}$ .

$$\text{Lowest required } f'_{\text{ci req}} = 5.19 / 0.65 = 7.98 \text{ ksi}$$

The bottom stress at release for this particular beam is essentially at the limit so the initial concrete strength cannot be reduced. If  $f'_{ci}$  could have been reduced, reanalysis of the beam would be necessary to ensure that stresses were still below the limits.

Proceed to the service and fatigue stress checks after final losses.

[5.9.2.3.2]

**4. Stresses at Service Loads (compression +, tension -)**

**Stress Limits for P/S Concrete after All Losses**

Compression in the concrete is limited to (Service I Load Combination):

$$f_{climf1} = 0.45 \cdot f'_c = 0.45 \cdot 9.5 = 4.28 \text{ ksi}$$

(for prestress and permanent loads)

Check the bottom stress at end of beam and the top stress at midspan against this limit.

$$f_{climf2} = 0.60 \cdot \phi_w \cdot f'_c = 0.60 \cdot 1.0 \cdot 9.5 = 5.70 \text{ ksi}$$

(for live load, prestress, permanent loads, and transient loads)

Check the top stress at midspan against this limit.

[5.5.3.1]

Compression in concrete is limited to (Fatigue I Load Combination):

$$f_{climfat} = 0.40 \cdot f'_c = 0.40 \cdot 9.5 = 3.80 \text{ ksi}$$

(for live load and 1/2 of prestress and permanent loads)

Check the top stress at midspan against this limit.

Tension in the concrete is limited to (Service III Load Combination):

$$f_{limf} = -0.19 \cdot \sqrt{f'_c} = -0.19 \cdot \sqrt{9.5} = -0.586 \text{ ksi}$$

Check the bottom stress at midspan against this limit.

**Check Stresses at Midspan After Losses:**

Bottom stress

$$\begin{aligned} &= -\left(\frac{M_{DC1}}{S_{gb}}\right) - \left(\frac{M_{DC2}}{S_{cb}}\right) - \left(\frac{M_{LL} \cdot 0.8}{S_{cb}}\right) + \left(\frac{P_e}{A_g}\right) + \left(\frac{P_e \cdot e_{strand}}{S_{gb}}\right) \\ &= -\left(\frac{2817 \cdot 12}{8246}\right) - \left(\frac{496 \cdot 12}{12,917}\right) - \left(\frac{1918 \cdot 12 \cdot 0.8}{12,917}\right) + \left(\frac{1773}{704}\right) + \left(\frac{1773 \cdot 13.83}{8246}\right) \\ &= -0.494 \text{ ksi} \end{aligned}$$

$$f_{limf} = -0.586 \text{ ksi} \quad \text{OK}$$

Top stress due to all loads

$$\begin{aligned} &= \left(\frac{P_e}{A_g}\right) - \left(\frac{P_e \cdot e_{strand}}{S_{gt}}\right) + \left(\frac{M_{DC1}}{S_{gt}}\right) + \left(\frac{M_{DC2} + M_{LL}}{S_{gtc}}\right) \\ &= \left(\frac{1773}{704}\right) - \left(\frac{1773 \cdot 13.83}{6794}\right) + \left(\frac{2817 \cdot 12}{6794}\right) + \left[\frac{(496 + 1918) \cdot 12}{42,761}\right] \\ &= 4.56 \text{ ksi} < 5.70 \text{ ksi} \quad \text{OK} \end{aligned}$$

Top stress due to permanent loads

$$= \left(\frac{P_e}{A_g}\right) - \left(\frac{P_e \cdot e_{strand}}{S_{gt}}\right) + \left(\frac{M_{DC1}}{S_{gt}}\right) + \left(\frac{M_{DC2}}{S_{gtc}}\right)$$



$$\begin{aligned}
 &= \left( \frac{1773}{704} \right) - \left( \frac{1773 \cdot 13.83}{6794} \right) + \left( \frac{2817 \cdot 12}{6794} \right) + \left( \frac{496 \cdot 12}{42,761} \right) \\
 &= 4.02 \text{ ksi} < 4.28 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

Top stress due to fatigue live load plus ½ the sum of prestress and permanent loads

$$\begin{aligned}
 &= \frac{1}{2} \left( \left( \frac{P_e}{A_g} \right) - \left( \frac{P_e \cdot e_{\text{strand}}}{S_{gt}} \right) + \left( \frac{M_{DC1}}{S_{gt}} \right) + \left( \frac{M_{DC2}}{S_{gtc}} \right) \right) + \left( \frac{M_{LL}}{S_{gtc}} \right) \\
 &= \frac{1}{2} \left( \left( \frac{1773}{704} \right) - \left( \frac{1773 \cdot 13.83}{6794} \right) + \left( \frac{2817 \cdot 12}{6794} \right) + \left( \frac{496 \cdot 12}{42,761} \right) \right) + \left( \frac{1008 \cdot 12}{42,761} \right) \\
 &= 2.29 \text{ ksi} < 3.80 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

### Check the Compression Stresses at End of Beam After Losses

Bottom flange stress at the transfer point due to prestress and permanent loads.

$$\begin{aligned}
 &= \frac{P_e}{A_g} + \frac{P_e \cdot e_{\text{strand}}}{S_{gb}} - \left( \frac{M_{DC1}}{S_{gb}} \right) - \left( \frac{M_{DC2}}{S_{gbc}} \right) \\
 &= \frac{1773}{704} + \frac{1773 \cdot 9.78}{8246} - \left( \frac{222 \cdot 12}{8246} \right) - \left( \frac{39 \cdot 12}{12,917} \right) \\
 &= 4.26 \text{ ksi} < 4.28 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

The final concrete strength,  $f'_c$  was assumed to be 9.5 ksi. For the most economical beam, the designer should choose the lowest required  $f'_c$  for the beam. This is determined by substituting the calculated maximum compression stress for  $f_{climf1}$  in the compression limit equation and solving for  $f'_c$ .

$$\text{Lowest required } f'_c = \frac{4.26}{0.45} = 9.47 \text{ ksi}$$

The assumed concrete strength cannot be reduced.

Keep  $f'_c = 9.5 \text{ ksi}$

### [5.5.4]

### 5. Flexure – Strength Limit State

Resistance factors at the strength limit state are:

$\phi = 1.00$  for flexure and tension (assumed)

$\phi = 0.90$  for shear and torsion

$\phi = 1.00$  for tension in steel in anchorage zones

Strength I design moment,  $M_u$ , is 7498 kip-ft at midspan.

From previous calculations, distance to strand centroid from bottom of the beam at midspan is:

$$y_{\text{strand}} = 4.24 \text{ in}$$

Similar to Grade 270 strands, the yield strength,  $f_{py}$  is taken as  $0.9 \cdot f_{pu}$ .

$$f_{py} = 0.9 \cdot f_{pu} = 0.9 \cdot 300 = 270 \text{ ksi}$$

**[5.6.3.1.1]**

$$k = 2 \cdot \left( 1.04 - \frac{f_{py}}{f_{pu}} \right) = 2 \cdot \left( 1.04 - \frac{270}{300} \right) = 0.280$$

$$\begin{aligned} d_p &= (\text{beam height}) + \text{stool} + \text{deck} - y_{\text{strand}} \\ &= 40 + 1.5 + 8.5 - 4.24 = 45.76 \text{ in} \end{aligned}$$

Begin by assuming the neutral axis lies in the deck.

For  $f'_c = 4.0$  ksi,  $\beta_1 = 0.85$  and  $\alpha_1 = 0.85$ .

Then

$$\begin{aligned} c &= \frac{A_{ps} \cdot f_{pu}}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_e + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}} \\ &= \frac{10.85 \cdot 300}{0.85 \cdot 4.0 \cdot 0.85 \cdot 82.00 + 0.28 \cdot 10.85 \cdot \left( \frac{300}{45.76} \right)} = 12.67 \text{ in} \end{aligned}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 12.67 = 10.77 \text{ in}$$

Compression block depth is greater than the thickness of the slab (8.5 in), so T-section behavior must be considered. The "web width",  $b_w$ , of the T-section is the beam flange width, which is 34 in.

Then

$$\begin{aligned} c &= \frac{A_{ps} \cdot f_{pu} - \alpha_1 \cdot f'_c \cdot (b - b_w) \cdot h_f}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_w + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}} \\ &= \frac{10.85 \cdot 300 - 0.85 \cdot 4.0 \cdot (82 - 34) \cdot 8.5}{0.85 \cdot 4.0 \cdot 0.85 \cdot 34 + 0.28 \cdot 10.85 \cdot \left( \frac{300}{45.76} \right)} = 15.81 \text{ in} \end{aligned}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 15.81 = 13.44 \text{ in}$$

The revised compression block depth is less than the thickness of the slab plus the flange thickness (15 in), so T-section behavior is confirmed. If the revised compression block depth had been greater than 15 inches, the section would be acting as a stepped T-section and a strain compatibility approach would have been necessary.

$$f_{ps} = f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right) = 300 \cdot \left(1 - 0.28 \cdot \frac{15.81}{45.76}\right) = 271.0 \text{ ksi}$$

The internal lever arm between compression and tension flexural force components is:

$$d_p - \frac{a}{2} = 45.76 - \frac{13.44}{2} = 39.04 \text{ in}$$

Then:

$$\begin{aligned} M_n &= A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + \alpha_1 \cdot f'_c \cdot (b - b_w) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right) \\ &= 10.85 \cdot 271.0 \cdot 39.04 + 0.85 \cdot 4.0 \cdot (82 - 34) \cdot 8.5 \cdot \left(\frac{13.44}{2} - \frac{8.5}{2}\right) \\ &= 118,218 \text{ kip-in} = 9852 \text{ kip-ft} \end{aligned}$$

$$M_r = \phi M_n = 1.0 \cdot 9852 = 9852 \text{ kip-ft} > M_u = 7498 \text{ kip-ft} \quad \text{OK}$$

**[5.5.4.2]**

Validate the assumption of 1.0 for the resistance factor:

Concrete compression strain limit  $\epsilon_c = 0.003$

Reinforcement tension-controlled strain limit  $\epsilon_{ti} = 0.005$

Referring to LRFD Figure C5.6.2.1-1 and using similar triangles in the prestressing strand,  $\epsilon_t$ :

$$\epsilon_t = (d_t - c) \cdot \left(\frac{\epsilon_c}{c}\right) = (45.76 - 15.81) \cdot \left(\frac{0.003}{15.81}\right) = 0.0056 > 0.005$$

Therefore  $\phi = 1.0$ , which matches the assumption

**[5.6.3.3]**

**6. Minimum Reinforcement**

To prevent brittle failure, an adequate amount of reinforcement is required.

Check that the section can carry the smaller of:

- 1)  $1.33M_u$
- 2) Cracking Moment,  $M_{cr}$

At midspan,  $1.33M_u = 1.33 \cdot 7498 = 9972 \text{ kip-ft}$

Lightweight concrete is not being used, so concrete density factor  $\lambda = 1.0$ .

$$f_r = 0.24 \cdot \lambda \cdot \sqrt{f'_c} = 0.24 \cdot 1.0 \cdot \sqrt{9.5} = 0.74 \text{ ksi}$$

$$\begin{aligned} f_{cpe} = f_{peb} &= \frac{P_e}{A_g} + \frac{P_e \cdot e_{strand}}{S_{gb}} \\ &= \frac{1773}{704} + \frac{1773 \cdot 13.83}{8246} = 5.49 \text{ ksi} \end{aligned}$$

$$M_{cr} = \gamma_3 \cdot \left[ (\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}) \cdot S_{gbc} - M_{dnc} \cdot \left(\frac{S_{gbc}}{S_{gb}} - 1\right) \right]$$

$$= 1.0 \cdot \left[ (1.6 \cdot 0.74 + 1.1 \cdot 5.49) \cdot 12,917 - 2817 \cdot 12 \cdot \left( \frac{12,917}{8246} - 1 \right) \right]$$

$$= 74,151 \text{ kip-in} = 6179 \text{ kip-ft} < 9972 \text{ kip-ft} \quad M_{cr} \text{ GOVERNS}$$

$$M_r = \phi M_n = 9852 \text{ kip-ft} > 6179 \text{ kip-ft} \quad \text{OK}$$

**F. Design  
Reinforcement for  
Shear**

[5.7]

[5.7.3.2]

**1. Vertical Shear Design**

**Determine  $d_v$  and Critical Section for Shear**

Begin by determining the effective shear depth  $d_v$  at the critical section for shear. The critical location for shear  $x_{vcrit}$  is defined as the distance  $d_v$  from the internal face of support. The internal face is assumed to be at the inside edge of the 15 inch long sole plate.

The effective shear depth is taken as the greatest of:

[5.7.2.8]

$$d_v = d_p - \frac{a}{2} \quad \text{or} \quad 0.72h_{comp} \quad \text{or} \quad 0.9d_e$$

[5.7.3.4.2]

For beams with draped strands, AASHTO is unclear on which strands to consider when determining  $d_p$  for calculating  $d_v$ . Considering LRFD Figure C5.7.2.8-1, it appears that  $d_v$  is based on calculating  $d_p$  and  $d_e$  for the strands found on the flexural tension side of the neutral axis. But for shear calculations, LRFD Article 5.7.3.4.2 and Figure 5.7.3.4.2-1 define  $A_{ps}$  as the strands found on the flexural tension side of  $\frac{1}{2}$  the height of the composite section. To keep computations simple, yet reasonably accurate, follow the LRFD Article 5.7.3.4.2 definition and consider only the prestressing strands found below  $\frac{1}{2}$  the height of the composite section when calculating  $d_p$ . Therefore, only straight prestressing strands are considered.

The flexural tension side of the member is defined as:

$$\frac{h_{comp}}{2} = \frac{50}{2} = 25 \text{ in}$$

The centroid of the straight prestressing strands is at:

$$y_{sstr} = \left[ \frac{(18 \cdot 2) + (14 \cdot 4) + (8 \cdot 6) + (2 \cdot 8)}{42} \right]$$

$$= 3.71 \text{ in}$$

With this strand centroid,  $d_p$  can be computed for the composite section:

$$d_p = h_{comp} - y_{sstr} = 50 - 3.71 = 46.29 \text{ in}$$

Recalculate the value of the compression block depth "a" considering only the straight prestressing strands:

$$A_{ps} = A_{sps} = (\# \text{ of straight strands}) \cdot (\text{strand area}) = 42 \cdot 0.217 = 9.11 \text{ in}^2$$

Begin by assuming the neutral axis lies in the deck.

[5.6.3.1.1]

$$c = \frac{A_{ps} \cdot f_{pu}}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_e + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}}$$

$$= \frac{9.11 \cdot 300}{0.85 \cdot 4.0 \cdot 0.85 \cdot 82.00 + 0.28 \cdot 9.11 \cdot \left(\frac{300}{46.29}\right)} = 10.78 \text{ in}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 10.78 = 9.16 \text{ in}$$

Compression block depth is greater than 8.5", the thickness of the slab, so T-section behavior must be considered.

$$c = \frac{A_{ps} \cdot f_{pu} - \alpha_1 \cdot f'_c \cdot (b - b_w) \cdot h_f}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_w + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}}$$

$$= \frac{9.11 \cdot 300 - 0.85 \cdot 4.0 \cdot (82 - 34) \cdot 8.5}{0.85 \cdot 4.0 \cdot 0.85 \cdot 34 + 0.28 \cdot 9.11 \cdot \left(\frac{300}{46.29}\right)} = 11.72 \text{ in}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 11.72 = 9.96 \text{ in}$$

$$d_v = d_p - \frac{a}{2} = 46.29 - \frac{9.96}{2} = 41.31 \text{ in}$$

But the effective shear depth  $d_v$  need not be less than

$$d_v \geq 0.72 \cdot h_{\text{comp}} = 0.72 \cdot 50 = 36.0 \text{ in}$$

or

$$d_v \geq 0.9 d_e = 0.9 d_p = 0.9 (46.29) = 41.7 \text{ in}$$

Take  $d_v = 41.7$  inches

Then the critical section for shear  $x_{\text{crit}}$  is:

$$x_{\text{crit}} = (0.5 \cdot \text{sole plate length}) + d_v$$

$$= (0.5 \cdot 15.0) + 41.7$$

$$= 49.2 \text{ in} = 4.1 \text{ ft from centerline of bearing}$$

**Check Maximum Factored Shear Limit**

From Table 5.7.2.4 the Strength I design shear at 4.1 ft is

$$V_u = 285 \text{ kips}$$

The amount of force carried by the draped strands at their effective prestress level is:

$$P_{8d} = 8 \cdot 0.217 \cdot 163.4 = 283.7 \text{ kips}$$

The inclination of the draped strands is:

$$\phi = \arctan \left[ \frac{(34 - 7) / 12}{47.83} \right] = 2.69 \text{ degrees}$$

The vertical prestress component is:

$$V_p = P_{8d} \cdot \sin(\phi) = 283.7 \cdot \sin(2.69) = 13.3 \text{ kips}$$

The superstructure is supported by an integral type abutment. Therefore, the nominal shear capacity of the section is limited to:

**[5.7.3.3]**

$$V_n = 0.25 \cdot f'_c \cdot d_v \cdot b_v + V_p = 0.25 \cdot 9.5 \cdot 41.7 \cdot 6.5 + 13.3 = 657 \text{ kips}$$

The maximum design shear the section can have is:

$$\phi_v \cdot V_n = 0.90 \cdot 657 = 591 \text{ kips} > 285 \text{ kips} \quad \text{OK}$$

Note that if the superstructure was supported by a parapet type abutment, which is not built integrally with its support, the shear stress would have been limited to  $0.18f'_c$  per AASHTO Article 5.7.3.2. Using an integral or semi-integral abutment allows us to use the higher value from AASHTO Article 5.7.3.3.

**Determine Longitudinal Strain  $\epsilon_s$** 

Assume that minimum transverse reinforcement will be provided in the cross section. As previously noted,  $A_{ps}$  includes only the area of prestressing steel found on the flexural tension side of the member, as defined in Figure 5.7.3.4.2-1. At  $x_{vcrit}$ ,  $A_{ps}$  consists of only the straight strands.

Near the end of the beam,  $A_{ps}$  must also be reduced for development, so  $f_{ps}$  must be calculated again for the end section following the process shown previously:

$$\begin{aligned} f_{ps} &= f_{pu} \cdot \left( 1 - k \cdot \frac{c}{d_p} \right) = 300 \cdot \left( 1 - 0.28 \cdot \frac{11.72}{46.29} \right) \\ &= 278.7 \text{ ksi} \end{aligned}$$

[5.9.4.3]

Development length  $\ell_d$  is:

$$\begin{aligned} \ell_d &= \kappa \cdot \left( f_{ps} - \frac{2}{3} \cdot f_{pe} \right) \cdot d_b \\ &= 1.6 \left[ 278.7 - \frac{2}{3} (163.4) \right] (0.6) = 163.0 \text{ in} \end{aligned}$$

Transfer length  $\ell_{tr}$  is:

$$\ell_{tr} = 60 \cdot d_b = 60 (0.6) = 36.0 \text{ in}$$

At the critical section  $x_{v\text{crite}} = (49.2 + 7.5) = 56.7$  in from the beam end, the strand development fraction is:

$$\begin{aligned} F_{\text{dev}} &= \frac{f_{pe}}{f_{ps}} + \frac{x_{v\text{crite}} - \ell_{tr}}{\ell_d - \ell_{tr}} \left( 1 - \frac{f_{pe}}{f_{ps}} \right) \\ &= \frac{163.4}{278.7} + \frac{56.7 - 36.0}{163.0 - 36.0} \left( 1 - \frac{163.4}{278.7} \right) = 0.65 \end{aligned}$$

Therefore,  $A_{ps} = A_{sps} \cdot F_{\text{dev}}$

$$= 9.11 \cdot 0.65 = 5.92 \text{ in}^2$$

[5.7.3.4.2]

Use LRFD equation 5.7.3.4.2-4 to compute the strain:

$$\begin{aligned} \epsilon_s &= \frac{\left( \left| \frac{M_u}{d_v} \right| + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po} \right)}{E_s \cdot A_s + E_p \cdot A_{ps}} \\ &= \frac{\left[ \left| \frac{1025 \cdot 12}{41.7} \right| + |285 - 13.3| - (5.92 \cdot 0.70 \cdot 300) \right]}{28,500 \cdot 5.92} = -0.00401 \end{aligned}$$

Because the value is negative, the strain will be recalculated using an additional concrete term:

From Figure 5.4.6.1 of this manual,  $A_{ct} = 435 \text{ in}^2$

$$\begin{aligned} \epsilon_s &= \frac{\left( \left| \frac{M_u}{d_v} \right| + 0.5N_u + |V_u - V_p| - A_{ps} \cdot f_{po} \right)}{E_c \cdot A_{ct} + E_s \cdot A_s + E_p \cdot A_{ps}} \\ &= \frac{\left[ \left| \frac{1025 \cdot 12}{41.7} \right| + |285 - 13.3| - (5.92 \cdot 0.70 \cdot 300) \right]}{4899 \cdot 435 + 28,500 \cdot 5.92} = -0.00029 \end{aligned}$$

Computed strain limits:

$$-0.0004 < -0.00029 < 0.006 \quad \text{OK}$$

Compute the tensile stress factor  $\beta$  using equation 5.7.3.4.2-1

$$\beta = \frac{4.8}{1 + 750 \cdot \epsilon_s} = \frac{4.8}{1 + 750 \cdot (-0.00029)} = 6.13$$

Compute the angle  $\theta$  using equation 5.7.3.4.2-3

$$\theta = 29 + 3500\epsilon_s = 29 + 3500 \cdot (-0.00029) = 28.0 \text{ degrees}$$

Compute the concrete contribution:

$$V_c = 0.0316 \cdot \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v = 0.0316 \cdot 6.13 \cdot \sqrt{9.5} \cdot 6.5 \cdot 41.7 = 161.8 \text{ kips}$$

The required steel contribution is

$$V_s = V_n - V_c - V_p = \frac{V_u}{\phi_v} - V_c - V_p = \frac{285}{0.90} - 161.8 - 13.3 = 141.6 \text{ kips}$$

Find the required spacing of double leg #4 stirrups:

$$s = \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_s} = \frac{2 \cdot 0.20 \cdot 60 \cdot 41.7 \cdot \cot(28.0)}{141.6} = 13.3 \text{ in}$$

Try double leg stirrups at a 12 inch spacing near the end of the beam.

$$A_v = \frac{0.4 \cdot 12}{12} = 0.40 \text{ in}^2 / \text{ft} \quad V_s = 156.9 \text{ kips}$$

### [5.7.2.5]

Check that the minimum transverse reinforcement requirement is satisfied:

$$\begin{aligned} \frac{A_{vmin}}{s} &= 0.0316 \cdot \lambda \cdot \sqrt{f'_c} \cdot \frac{b_v}{f_y} \\ &= 0.0316 \cdot 1.0 \cdot \sqrt{9.5} \cdot \frac{6.5}{60} \cdot 12 = 0.13 \frac{\text{in}^2}{\text{ft}} < 0.40 \frac{\text{in}^2}{\text{ft}} \quad \text{OK} \end{aligned}$$

### [5.7.2.6]

Check maximum permitted stirrup spacing at  $X_{vcrite}$ :



$$v_u = \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v} = \frac{285 - 0.90 \cdot 13.3}{0.90 \cdot 6.5 \cdot 41.7} = 1.12 \text{ ksi}$$

$$v_{u\text{limit}} = 0.125 \cdot f'_c = 0.125 \cdot 9.5 = 1.19 \text{ ksi} > 1.12 \text{ ksi}$$

Then the maximum spacing is the smaller of:

$$s_{\text{max}} = 0.8 \cdot d_v = 0.8 \cdot 41.7 = 33.0 \text{ in}$$

$$\text{or } s_{\text{max}} = 24 \text{ in} \quad \text{GOVERNS}$$

$$s_{\text{max}} = 24 \text{ in} > 12 \text{ in} \quad \text{OK}$$

Therefore, use double leg #4 stirrups at 12 inch spacing. Other sections are investigated similarly.

#### [5.7.4]

#### 2. Interface Shear Transfer

The standard beam details require that the outer 6 inches on each edge of the top flange will be smooth with a bond breaker applied, which leaves 22 inches of the top flange to be roughened for engagement of shear transfer.

Then  $b_{vi} = 22 \text{ in}$

The Strength I vertical shear at the critical shear section due to all loads is:

$$V_u = 285 \text{ kip}$$

$$v_{ui} = \frac{V_u}{b_{vi} \cdot d_v} = \frac{285}{22 \cdot 41.7} = 0.31 \text{ ksi}$$

Interface shear force is:

$$V_{ui} = v_{ui} \cdot \frac{12 \text{ in}}{\text{ft}} \cdot b_{vi} = 0.31 \cdot 12 \cdot 22 = 81.8 \frac{\text{kips}}{\text{ft}}$$

Required nominal interface design shear is:

$$V_{n\text{ireg}} = \frac{V_{ui}}{\phi_v} = \frac{81.8}{0.90} = 90.9 \frac{\text{kips}}{\text{ft}}$$

The interface area per 1 foot length of beam is:

$$A_{cv} = 22 \cdot 12 = 264.0 \text{ in}^2/\text{ft}$$

#### [5.7.4.4]

The standard beam details require the top flanges of the beam to be roughened. Then:

$$c = 0.28 \text{ ksi} \quad \mu = 1.0 \quad K_1 = 0.3 \quad K_2 = 1.8 \text{ ksi}$$

The upper limits on nominal interface shear are:

$$K_1 \cdot f'_c \cdot A_{cv} = 0.3 \cdot 4 \cdot 264.0 = 316.8 \text{ kip/ft} > 90.9 \text{ kip/ft} \quad \text{OK}$$

and

$$K_2 \cdot A_{cv} = 1.8 \cdot 264.0 = 475.2 \text{ kip/ft} > 90.9 \text{ kip/ft} \quad \text{OK}$$

The nominal interface shear resistance is:

$$V_{ni} = cA_{cv} + \mu(A_{vf} \cdot f_y + P_c)$$

$$P_c = 0.0 \text{ kip}$$

Substitute and solve for required interface shear steel:

$$A_{vfreq} = \frac{V_{nireq} - c \cdot A_{cv}}{\mu \cdot f_y} = \frac{90.9 - 0.28 \cdot 264.0}{1.0 \cdot 60} = 0.28 \text{ in}^2/\text{ft}$$

#### [5.7.4.2]

Check minimum interface shear requirements:

The minimum requirement may be waived for girder-slab interfaces with the surface roughened to an amplitude of 0.25 in if the factored interface shear stress is less than 0.210 ksi.

$$v_{ui} = 0.31 \text{ ksi} > 0.210 \text{ ksi}$$

Then the minimum requirement cannot be waived.

The minimum required interface shear reinforcement is the lesser of:

$$A_{vfmin1} = \frac{0.05 \cdot b_v}{f_y} = \frac{0.05 \cdot 22}{60} = 0.018 \text{ in}^2/\text{in} = 0.22 \text{ in}^2/\text{ft}$$

or

$$\begin{aligned} A_{vfmin2} &= \frac{1.33 \cdot V_{nireq} - c \cdot A_{cv}}{\mu \cdot f_y} = \frac{1.33 \cdot 90.9 - 0.28 \cdot 264}{1.0 \cdot 60} \\ &= 0.78 \text{ in}^2/\text{ft} \end{aligned}$$

$$\text{Then } A_{vfmin} = 0.22 \text{ in}^2/\text{ft}$$

The double leg #4 stirrup at 12" spacing ( $A_v=0.40 \text{ in}^2/\text{ft}$ ) chosen earlier for vertical shear also meets the requirements for interface shear. Therefore, no additional reinforcement is required for interface shear.

Other sections are investigated similarly.

**[5.7.3.5]****3. Minimum Longitudinal Reinforcement Requirement**

The longitudinal reinforcement must be checked to ensure it is adequate to carry the tension caused by shear. The amount of strand development must be considered near the end of the beam. There are 2 cases to be checked:

Case 1: From the inside edge of bearing at the end supports out to the critical section for shear, the following must be satisfied, with  $A_{ps} \cdot f_{ps}$  modified for development and  $V_p$  modified for amount of prestress transfer:

$$A_{ps} \cdot f_{ps} \geq \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot(\theta)$$

A crack starting at the inside edge of the bearing sole plate will cross the center of gravity of the straight strands at:

$$x_{\text{crack}} = L_{\text{soleplate}} + y_{\text{sstr}} \cdot \cot(\theta) = 15 + 3.71 \cdot \cot(28.0) = 22.0 \text{ in}$$

The transfer length for 0.6" strands is:  $\ell_{tr} = 36.0 \text{ in}$

From the end of the beam to full transfer length, the strand stress increases linearly from zero to  $f_{pe}$ . Interpolate to find the tensile capacity of the straight strands at the intersection with the assumed crack:

$$T_{r1} = A_{ps} \cdot f_{pe} \cdot \frac{x_{\text{crack}}}{\ell_{tr}} = 42 \cdot 0.217 \cdot 163.4 \cdot \frac{22.0}{36} = 910 \text{ kips}$$

The prestress component in the direction of the shear force must be reduced because  $x_{\text{crack}} < \ell_{tr}$ :

$$V_{\text{pred}} = V_p \cdot \frac{x_{\text{crack}}}{\ell_{tr}} = 13.3 \cdot \frac{22.0}{36.0} = 8.1 \text{ kips}$$

The tension force to carry is:

$$\begin{aligned} T_{u1} &= \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_{\text{pred}} \right) \cdot \cot(\theta) \\ &= \left( \frac{285}{0.90} - 0.5 \cdot 156.9 - 8.1 \right) \cdot \cot(28.0) \\ &= 432.8 \text{ kips} < 910 \text{ kips} \quad \text{OK} \end{aligned}$$

Case 2: At the critical section for shear, the following must be satisfied, with  $A_{ps} \cdot f_{ps}$  modified for development:

$$A_{ps} \cdot f_{ps} \geq \frac{M_u}{\phi_f d_v} + \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot(\theta)$$

Use values calculated earlier to determine the tensile capacity at the critical section for shear:

$$f_{ps} = 278.7 \text{ ksi}$$

$$A_{ps} = 9.11 \text{ in}^2$$

$$\text{Development fraction, } F_{dev} = 0.65$$

$$T_{r2} = A_{ps} \cdot f_{ps} \cdot F_{dev} = 9.11 \cdot 278.7 \cdot 0.65 = 1650 \text{ kips}$$

The factored moment  $M_u$  should be the moment concurrent with the factored shear  $V_u$  at  $x_{vcrit}$ . For simplicity, the maximum  $M_u$  at  $x_{vcrit}$  is used below.

Modification of  $V_p$  is not required because  $x_{vcrit} > l_{tr}$ .

Then the tension force to carry is:

$$\begin{aligned} T_{u2} &= \frac{M_u}{\phi_f d_v} + \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot(\theta) \\ &= \frac{1025 \cdot 12}{1.0 \cdot 41.7} + \left( \frac{285}{0.9} - 0.5 \cdot 156.9 - 13.3 \right) \cdot \cot(28.0) \end{aligned}$$

$$T_{u2} = 718.0 \text{ kips} < 1650 \text{ kips} \quad \text{OK}$$

**G. Design  
Pretensioned  
Anchorage Zone  
Reinforcement  
[5.9.4.4.1]**

**Splitting Reinforcement**

To prevent cracking in the beam end due to the transfer of the prestressing force from the strands to the concrete, splitting reinforcement needs to be provided in the anchorage zone.

Use a load factor of 1.0 and lateral force component of 4% to determine the required amount of steel.

The total prestressing force at transfer:

$$P_i = A_{ps} \cdot (f_{pj} - \Delta f_{pES}) = 10.85 \cdot (216.0 - 25.6) = 2066 \text{ kips}$$

The factored design splitting force is:

$$P_{split} = 1.0 \cdot 0.04 \cdot P_i = 1.0 \cdot 0.04 \cdot 2066 = 82.6 \text{ kips}$$

The amount of resisting reinforcement is determined using a steel stress  $f_s$  of 20 ksi:

$$A_s = \frac{P_{split}}{f_s} = \frac{82.6}{20} = 4.13 \text{ in}^2$$

This steel should be located at the end of the beam within a distance of:

$$\frac{h}{4} = \frac{40}{4} = 10 \text{ in}$$

The number of #5 double legged stirrups necessary to provide this area is:

$$\frac{A_s}{2 \cdot A_b} = \frac{4.13}{2 \cdot 0.31} = 6.7$$

The first set of stirrups (G505E) is located 2 inches from the end of the beam. See Figure 5.7.2.5.

Provide an additional six sets of #5 stirrups (G508E) spaced at 2 1/2 inch centers.

$$x_{\text{splitting}} = 2 + 6 \cdot 2.5 = 17.0 \text{ in} > 10 \text{ in}$$

Although the splitting reinforcement does not fit within  $h/4$ , #5 bars are the largest allowed and 2.5 inches is the tightest spacing allowed. This is OK per MnDOT practice.

#### [5.9.4.4.2]

#### Confinement Reinforcement

Reinforcement is required at the ends of the beam to confine the prestressing steel in the bottom flange. G303E bars (see Figure 5.7.2.5) will be placed at a maximum spacing of 6 inches out to 1.5d from the ends of the beam. For simplicity in detailing and ease of tying the reinforcement, space the vertical shear reinforcement with the confinement reinforcement in this area.

$$1.5 d = 1.5 \cdot 40 = 60.0 \text{ in}$$

#### H. Determine Camber and Deflection

##### [2.5.2.6.2]

##### [3.6.1.3.2]

##### [5.6.3.5.2]

#### Camber Due to Prestressing and Dead Load Deflection

Using the PCI handbook (Figure 4.10.13 of the 3<sup>rd</sup> Edition), the camber due to prestress can be found. The centroid of the prestressing has an eccentricity  $e_{\text{mid}}$  of 13.83 inches at midspan. At the end of the beams the eccentricity  $e_e$  is 9.51 inches.  $E$  is the initial concrete modulus (4578 ksi),  $P_o$  equals the prestress force just after transfer (2066 kips). The drape points are at 0.4 of the design span, which is 118.0 feet. The span length at release is the end-to-end length of the 119.25 feet since the beam is in the casting bed. Using the equation for the two-point depressed strand pattern:

$$e' = e_{\text{mid}} - e_e = 13.83 - 9.51 = 4.32 \text{ in}$$

$$\Delta_{\text{ps}} = \frac{P_o \cdot e_e \cdot L^2}{8 \cdot E \cdot I} + \frac{P_o \cdot e'}{E \cdot I} \left( \frac{L^2}{8} - \frac{a^2}{6} \right)$$

$$\begin{aligned}
 &= \frac{2066 \cdot 9.51 \cdot (119.25 \cdot 12)^2}{8 \cdot 4578 \cdot 149,002} \\
 &\quad + \frac{2066 \cdot 4.32}{4578 \cdot 149,002} \left[ \frac{(119.25 \cdot 12)^2}{8} - \frac{(0.4 \cdot 118 \cdot 12 + 7.50)^2}{6} \right] \\
 &= 10.00 \text{ in}
 \end{aligned}$$

Downward deflection due to selfweight

$$\Delta_{sw} = \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I} = \frac{5 \cdot \frac{0.758}{12} (119.25 \cdot 12)^4}{384 \cdot 4578 \cdot 149,002} = 5.06 \text{ in}$$

Camber at release  $\Delta_{rel} = \Delta_{ps} - \Delta_{sw} = 10.00 - 5.06 = 4.94 \text{ in}$

To estimate camber at the time of erection the deflection components are multiplied by standard MnDOT multipliers. They are:

Release to Erection Multipliers:

Prestress = 1.4

Selfweight = 1.4

Camber and selfweight deflection values at erection are:

Prestress:	$1.4 \cdot 10.00 = 14.00 \text{ in}$
Selfweight:	$1.4 \cdot (-5.06) = -7.08 \text{ in}$
Diaphragm DL:	-0.02 in
Deck and stool DL:	-5.12 in
Barrier:	-0.37 in

Note that the deflection values for diaphragms, deck, stool, and barrier are based on a span length of 118.0 feet.

The values to be placed in the camber diagram on the beam plan sheet are arrived at by combining the values above.

"Erection Camber" =  $14.00 - 7.08 - 0.02 = 6.90 \text{ in}$  say 6 7/8 in

"Est. Dead Load Deflection" =  $5.12 + 0.37 = 5.49 \text{ in}$  say 5 1/2 in

"Est. Residual Camber" =  $6 \frac{7}{8} - 5 \frac{1}{2} = 1 \frac{3}{8} \text{ in}$

### Live Load Deflection

The deflection of the bridge is checked when subjected to live load and compared against the limiting values of  $L/800$  for vehicle only bridges and  $L/1000$  for bridges with bicycle or pedestrian traffic.

Deflection due to lane load is:

$$\Delta_{\text{lane}} = \left( \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I} \right) = \left[ \frac{5 \cdot \frac{0.64}{12} \cdot (118 \cdot 12)^4}{384 \cdot 4899 \cdot 396,823} \right] = 1.44 \text{ in}$$

Deflection due to a truck with dynamic load allowance is found using hand computations or computer tools to be:

$$\Delta_{\text{truck}} = 2.81 \text{ in}$$

Two deflections are computed and compared to the limiting values, that of the truck alone and that of the lane load plus 25% of the truck. Both deflections need to be adjusted with the live load distribution factor for deflection.

$$\Delta_1 = DF_{\Delta} \cdot \Delta_{\text{truck}} = 0.425 \cdot 2.81 = 1.19 \text{ in}$$

$$\Delta_2 = DF_{\Delta} \cdot (\Delta_{\text{lane}} + 0.25 \cdot \Delta_{\text{truck}}) = 0.425 \cdot (1.44 + 0.25 \cdot 2.81) = 0.91 \text{ in}$$

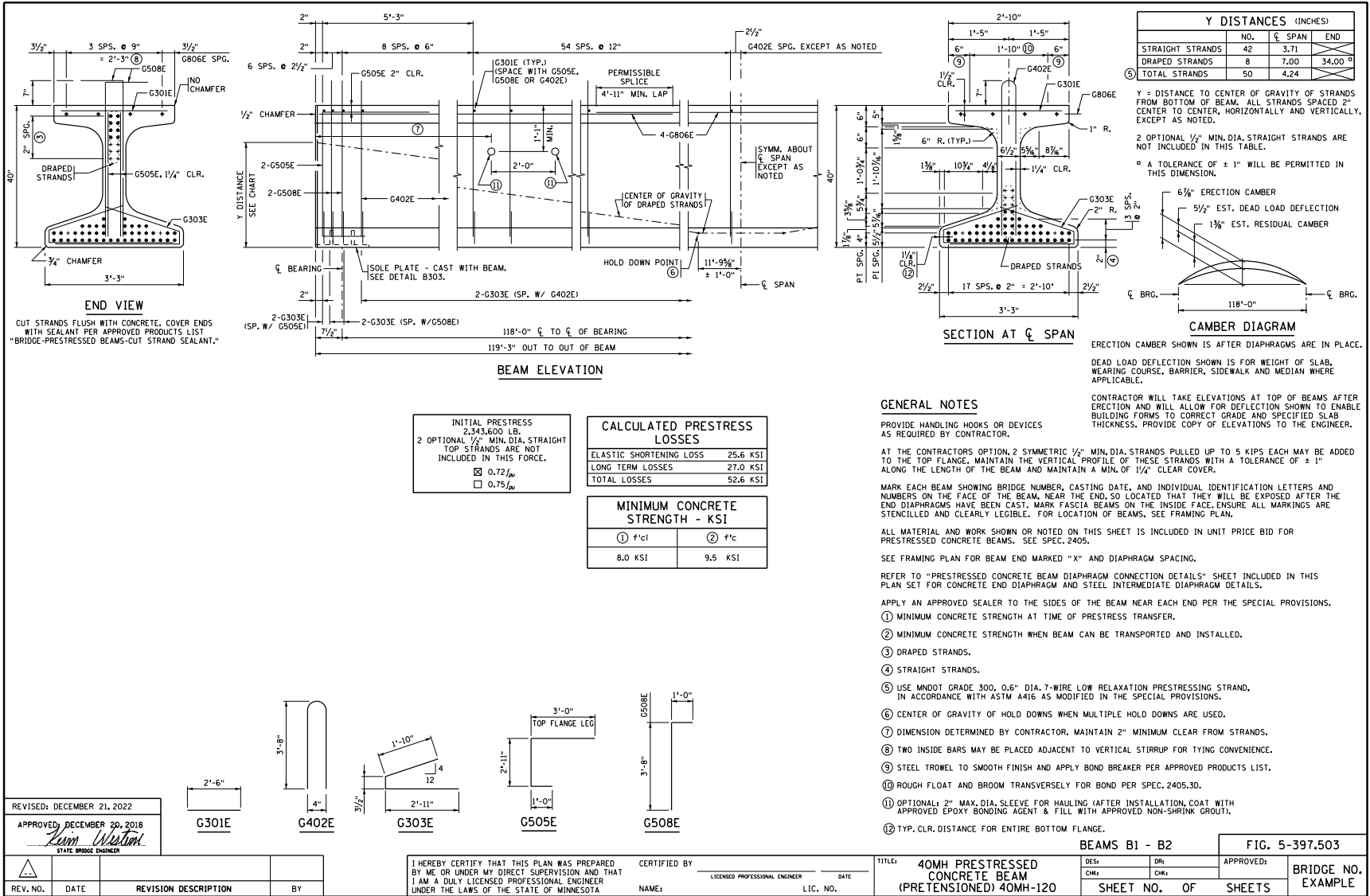
There is no bicycle or pedestrian traffic on the bridge, so the deflection limit is:

$$\frac{L}{800} = \frac{118 \cdot 12}{800} = 1.77 \text{ in} > \text{ than } \Delta_1 \text{ or } \Delta_2 \quad \text{OK}$$

### ***I. Beam Sheet for Bridge Plan***

Figure 5.7.2.5 shows the detailed beam sheet for the draped strand configuration that will be included in the bridge plan.

Figure 5.7.2.5

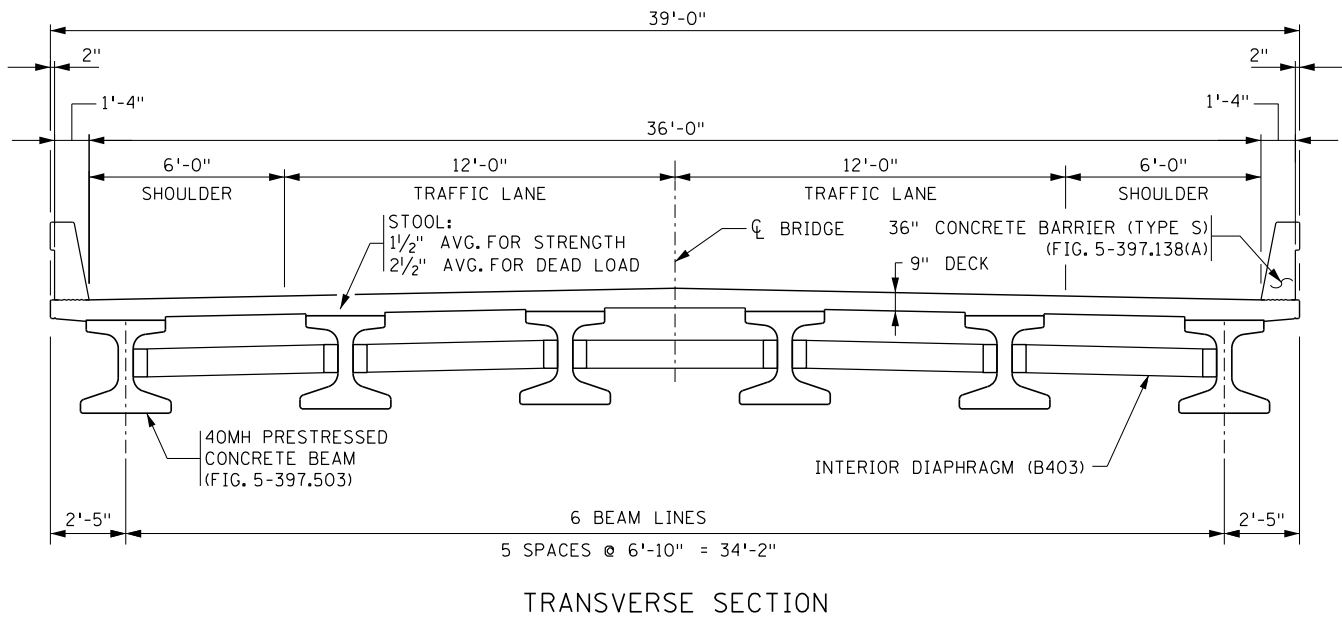




**5.7.3 Debonded  
Prestressed I-  
Beam Design  
Example**

This example illustrates the design of a pretensioned I-beam for a single span bridge without skew. The 118'-0" span is supported with MnDOT "40MH" beams on integral abutments. MnDOT standard details and drawings for diaphragms (B403), barriers (Fig. 5-397.138(A)), and beams (Fig. 5-397.503) are to be used with this example. This example contains the design of a typical interior beam at the critical sections in positive flexure, shear, and deflection. The superstructure consists of six beams spaced at 6'-10" centers. A typical transverse superstructure section is provided in Figure 5.7.3.1. A framing plan is provided in Figure 5.7.3.2. The roadway section is composed of two 12' traffic lanes and two 6' shoulders. A Type S barrier is provided on each side of the bridge and a 9" monolithic concrete deck is used. Interior diaphragms are used at the interior third points based on guidance found in BDM Table 5.4.1.1.

This example uses 0.6" diameter, 300 ksi, low relaxation strands for pretensioning. Debonded strands are used to control the beam end stresses.



**Figure 5.7.3.1**

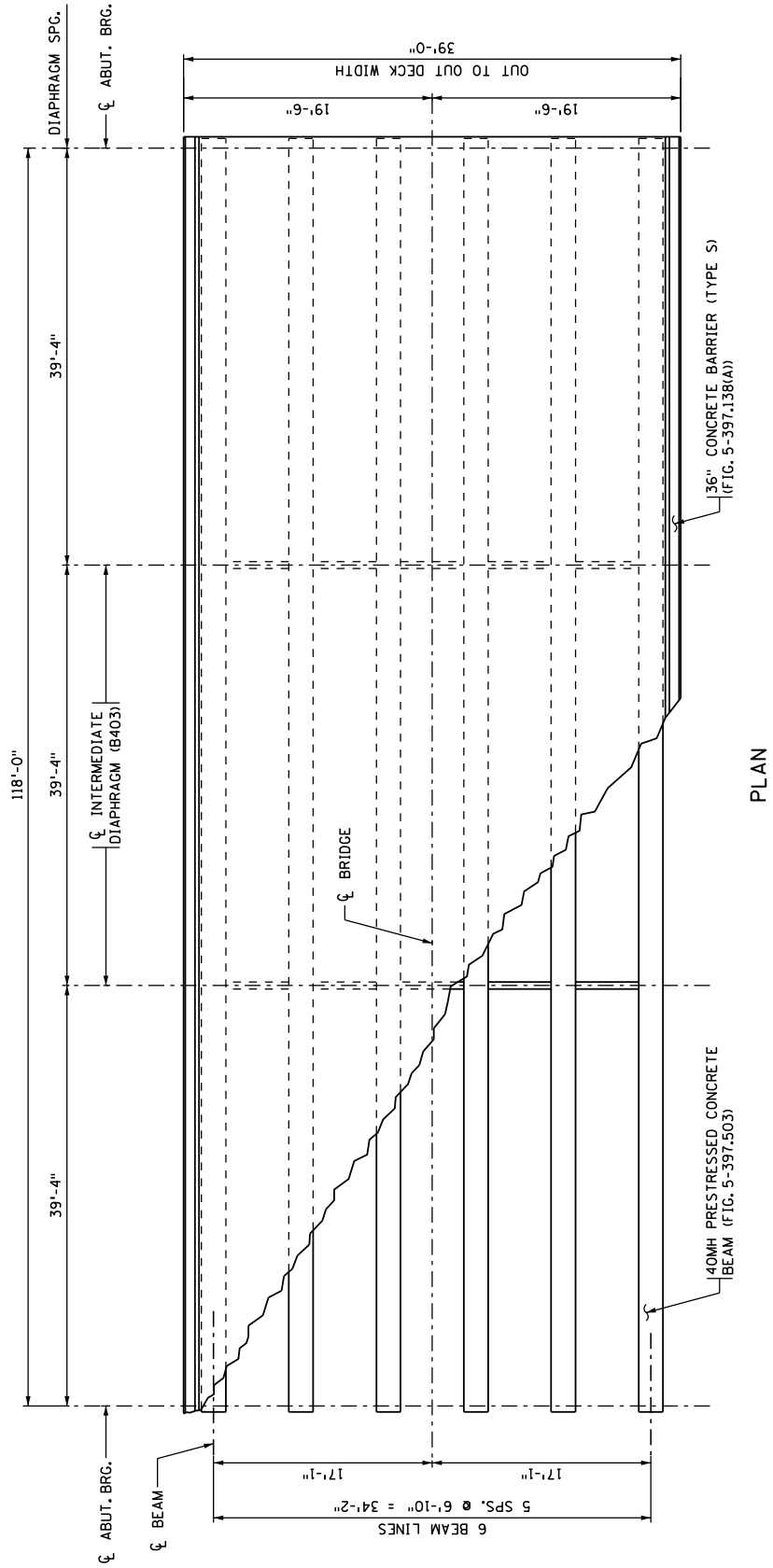


Figure 5.7.3.2

**A. Materials**

The modulus of elasticity for high strength concrete suggested by ACI Committee 363 is used for the beam concrete. AASHTO Article 5.4.2.4 is used to calculate  $E_c$  for the deck and assumes a  $K_1$  equal to 1.0. The composite deck is assumed to have a unit weight of 0.150 kcf for dead load computations and 0.145 kcf for  $E_c$  computations. The beam concrete is assumed to have a unit weight of 0.155 kcf for dead load computations.

The material and geometric parameters used in the example are shown in Table 5.7.3.1:

**Table 5.7.3.1  
Material Properties**

Material Parameter		Prestressed Beam	Deck
Concrete	$f'_{ci}$ at transfer	8.0 ksi *	---
	$f'_c$ at 28 days	9.5 ksi *	4 ksi
	$E_{ci}$ at transfer	$(1265 \cdot \sqrt{f'_{ci}}) + 1000$ = 4578 ksi	---
	$E_c$ at 28 days	$(1265 \cdot \sqrt{f'_c}) + 1000$ = 4899 ksi	$120,000 \cdot K_1 \cdot (w_c)^2 \cdot (f'_c)^{0.33}$ = 3987 ksi
Steel	$f_y$ for rebar	60 ksi	60 ksi
	$f_{pu}$ for strand	300 ksi	---
	$E_s$ for rebar	29,000 ksi	29,000 ksi
	$E_p$ for strand	28,500 ksi	---
	Strand type	0.6 inch diameter 300 ksi, low relaxation	---

\*These concrete compressive strength values are initial assumed values. Final values may differ based on adjustments for the actual and initial final service stresses.

**B. Determine Cross-Section Properties for a Typical Interior Beam**

The beams are designed to act compositely with the deck on simple spans. The deck consists of a 9 inch thick concrete slab. A 1/2 inch of wear is assumed. A thickness of 8 1/2 inches is used for composite section properties. The stool height is assumed to be an average of 2 1/2 inches for dead load computations and 1 1/2 inches for section property computations.

**[4.6.2.6.1]**

The effective flange width,  $b_e$ , is equal to the average beam spacing:

$$b_e = 82.00 \text{ in}$$

To transform the deck and stool concrete to beam concrete, use a modular ratio  $n_{d\_bm}$  based on  $E_{cdeck}$  to  $E_{cbeam}$ :

$$n_{d\_bm} = \frac{E_{cdeck}}{E_{cbeam}} = \frac{3987}{4899} = 0.81$$

This results in a transformed effective flange width of:

$$b_{\text{etrans}} = n_{d\_bm} \cdot b_e = 0.81 \cdot 82 = 66.42 \text{ in}$$

Properties for an interior beam are given in Table 5.7.3.2.

**Table 5.7.3.2**  
**Cross-Section Properties**

Parameter	Non-composite Section	Composite Section
Height of section, h	40.00 in	50.00 in
Deck thickness	---	8.50 in
Average stool thickness	---	1.50 in (section properties) 2.50 in (dead load)
Effective flange width, $b_e$	---	82.00 in (deck concrete) 66.42 in (beam concrete)
Area, A	704 in <sup>2</sup>	1310 in <sup>2</sup>
Moment of inertia, I	149,002 in <sup>4</sup>	396,823 in <sup>4</sup>
Centroidal axis height, y	18.07 in	30.72 in
Bottom section modulus, $S_b$	8246 in <sup>3</sup>	12,917 in <sup>3</sup>
Top section modulus, $S_t$	6794 in <sup>3</sup>	25,410 in <sup>3</sup>
Top of prestressed beam, $S_{tbm}$	6794 in <sup>3</sup>	42,761 in <sup>3</sup>

**C. Live Load**  
**Distribution Factors**  
**and Load Modifiers**

Assume that traffic can be positioned anywhere between the barriers.

$$\text{Number of design lanes} = \frac{\text{distance between barriers}}{\text{design lane width}} = \frac{36}{12} = 3$$

**[4.6.2.2]**

**1. Determine Live Load Distribution Factors**

Designers should note that the approximate live load distribution factor equations include the multiple presence factors.

**[4.6.2.2.2]**

**Live Load Distribution Factor for Moment – Interior Beams**

LRFD Table 4.6.2.2.1-1 lists the common deck superstructure types for which approximate live load distribution equations have been assembled. The cross section for this design example is Type (k). To ensure that the approximate distribution equations can be used, several parameters need to be checked.

- 1) 3.5 ft ≤ beam spacing = 6.83 ft ≤ 16.0 ft    OK
- 2) 4.5 in ≤ slab thickness = 8.5 in ≤ 12.0 in    OK
- 3) 20 ft ≤ span length = 118 ft ≤ 240 ft    OK
- 4) 4 ≤ number of beams = 6    OK

The live load distribution factor equations use a  $K_g$  factor that is defined in LRFD Article 4.6.2.2.1. For determination of  $K_g$ , the beam concrete is transformed to deck concrete, so the modular ratio  $n_{bm\_d}$  differs from  $n_{d\_bm}$  calculated earlier.

$$n_{bm\_d} = \frac{E_{cbeam}}{E_{cdeck}} = \frac{4899}{3987} = 1.23$$

$$e_g = (\text{deck centroid}) - (\text{beam centroid}) = 45.75 - 18.07 = 27.68 \text{ in}$$

$$K_g = n_{bm\_d} \cdot [I + A \cdot (e_g)^2] = 1.23 \cdot [149,002 + 704 \cdot (27.68)^2] \\ = 8.47 \times 10^5 \text{ in}^4$$

Check  $K_g$  limits:  $1 \times 10^4 \leq K_g = 8.47 \times 10^5 \leq 7 \times 10^6$  OK

For one design lane loaded, the live load distribution factor for moment,  $gM_{int\_1lane}$ , is:

$$gM_{int\_1lane} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K}{12 \cdot L \cdot t_s^3}\right)^{0.1}$$

$$gM_{int\_1lane} = 0.06 + \left(\frac{6.83}{14}\right)^{0.4} \cdot \left(\frac{6.83}{118}\right)^{0.3} \cdot \left(\frac{8.47 \times 10^5}{12 \cdot 118 \cdot 8.5^3}\right)^{0.1}$$

$$gM_{int\_1lane} = 0.378 \text{ lanes/beam}$$

Two or more design lanes loaded:

$$gM_{int\_mlane} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K}{12 \cdot L \cdot t_s^3}\right)^{0.1}$$

$$gM_{int\_mlane} = 0.075 + \left(\frac{6.83}{9.5}\right)^{0.6} \cdot \left(\frac{6.83}{118}\right)^{0.2} \cdot \left(\frac{8.47 \times 10^5}{12 \cdot 118 \cdot 8.5^3}\right)^{0.1}$$

$$gM_{int\_mlane} = 0.538 \text{ lanes/beam}$$

#### [4.6.2.2.2d]

#### Live Load Distribution Factor for Moment - Exterior Beams

LRFD Table 4.6.2.2.2d-1 contains the approximate live load distribution factor equations for exterior beams. Type (k) cross-sections have a deck dimension check to ensure that the approximate equations are valid. The distance from the inside face of barrier to the centerline of the fascia beam is defined as  $d_e$ . For the example this distance is:

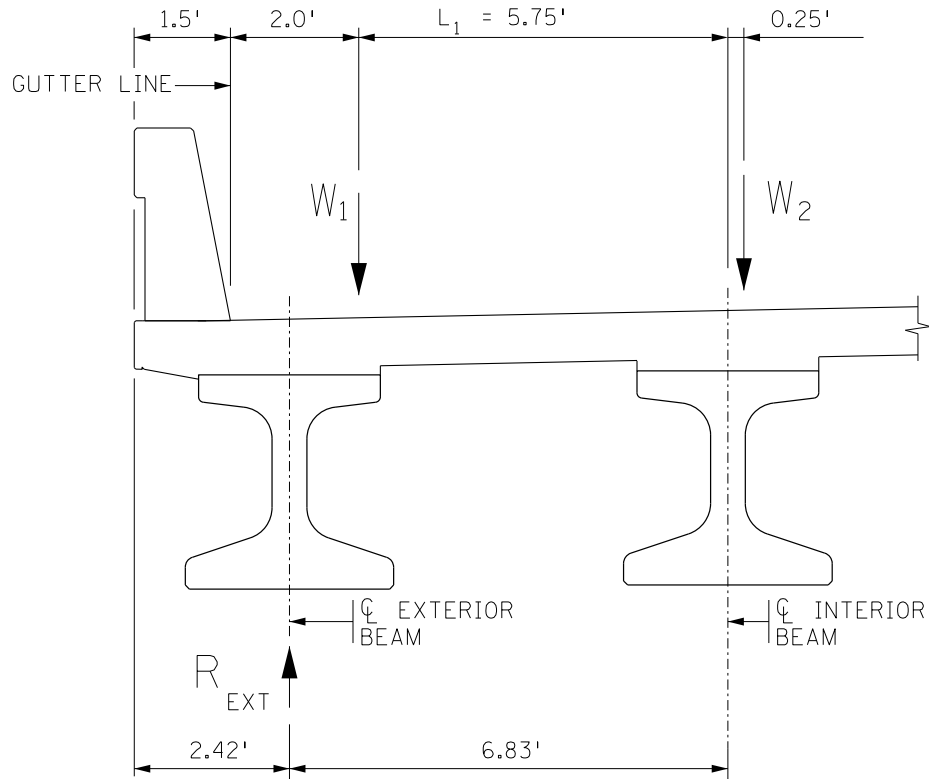
$$d_e = \text{deck overhang} - \text{deck coping} - \text{barrier width}$$

$$= \frac{(29 - 2 - 16)}{12} = 0.92 \text{ ft}$$

Check whether approximate equations can be used:

$$-1.0 \text{ ft} \leq d_e = 0.92 \text{ ft} \leq 5.5 \text{ ft} \quad \text{OK}$$

One design lane loaded:



**Figure 5.7.3.3**

Use the lever rule to determine the live load distribution factor for one lane. The exterior beam live load distribution factor is found by determining the exterior beam reaction and applying the multiple presence factor, *m*, for one lane:

**[Table 3.6.1.1.2-1]**

$$W_1 = W_2 = 0.5 \text{ lanes}$$

$$gM_{\text{ext}_1\text{lane}} = R_{\text{ext}} \cdot m = \left( \frac{W_1 \cdot L_1}{S} \right) \cdot m = \left( \frac{0.5 \cdot 5.75}{6.83} \right) \cdot 1.20$$

$$gM_{\text{ext}_1\text{lane}} = 0.505 \text{ lanes/beam}$$

Two or more design lanes loaded:

The live load distribution factor is equal to the factor “*e*” multiplied by the interior girder live load distribution factor for two or more lanes.

$$e = 0.77 + \left(\frac{d_e}{9.1}\right) = 0.77 + \left(\frac{0.92}{9.1}\right) = 0.871$$

$$gM_{\text{ext\_mlane}} = e \cdot gM_{\text{int\_mlane}} = 0.871 \cdot 0.538 = 0.469 \text{ lanes/beam}$$

**[4.6.2.2.2e]****Skew Factor**

No correction is necessary for a skew angle of zero.

**[4.6.2.2.3]****[4.6.2.2.3a]****Live Load Distribution Factor for Shear – Interior Beams**

LRFD Table 4.6.2.2.3a-1 can be used.

One design lane loaded:

$$gV_{\text{int\_1lane}} = 0.36 + \left(\frac{S}{25.0}\right) = 0.36 + \left(\frac{6.83}{25}\right) = 0.633 \text{ lanes/beam}$$

Two or more design lanes loaded:

$$gV_{\text{int\_mlane}} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^2 = 0.2 + \left(\frac{6.83}{12}\right) - \left(\frac{6.83}{35}\right)^2$$

$$= 0.731 \text{ lanes/beam}$$

**[4.6.2.2.3b]****Live Load Distribution Factor for Shear – Exterior Beams**

One Design Lane Loaded:

Use the lever rule, which results in the same factor that was computed for flexure and is equal to 0.505 lanes/beam

Two or more design lanes loaded:

$$e = 0.6 + \left(\frac{d_e}{10}\right) = 0.6 + \left(\frac{0.92}{10}\right) = 0.692$$

The exterior beam shear live load distribution factor for two or more design lanes is determined by modifying the interior distribution factor:

$$gV_{\text{ext\_mlane}} = e \cdot gV_{\text{int\_mlane}} = 0.692 \cdot 0.731 = 0.506 \text{ lanes/beam}$$

**[4.6.2.2.3c]****Skew Factor**

No correction is necessary for a skew angle of zero.

**[2.5.2.6.2]****[Table 3.6.1.1.2-1]****Live Load Distribution Factor for Deflection**

The live load distribution factor for checking live load deflections assumes that the entire cross section participates in resisting the live load. The deflection live load distribution factor is:

$$gD = \frac{(\# \text{ of lanes}) \cdot m}{(\# \text{ of beam lines})} = \frac{3 \cdot 0.85}{6} = 0.425 \text{ lanes/beam}$$

**Live load Distribution Factor for Fatigue – Interior and Exterior Beams**

[3.6.1.1.2]

The fatigue limit state is to be analyzed for one traffic lane, but the multi-presence factor does not apply. The live load distribution factor for one lane is to be divided by 1.2 to account for this.

Interior Beam:

$$g_{F_{int\_1lane}} = \frac{gM_{int\_1lane}}{1.2} = \frac{0.378}{1.2} = 0.315 \text{ lanes/beam}$$

Exterior Beam:

$$g_{F_{ext\_1lane}} = \frac{gM_{ext\_1lane}}{1.2} = \frac{0.505}{1.2} = 0.421 \text{ lanes/beam}$$

Table 5.7.3.3 contains a summary of the live load distribution factors and Table 5.7.3.4 contains a summary of the load modifiers for this example.

**Table 5.7.3.3**

**Live Load Distribution Factor Summary (lanes per beam)**

Loading		Flexure	Shear	Deflection	Fatigue
Interior Beam	One Design Lane	0.378	0.633	-	0.315
	Two or More Design Lanes	0.538	0.731	0.425	-
Exterior Beam	One Design Lane	0.505	0.505	-	0.421
	Two or More Design Lanes	0.469	0.506	0.425	-

[1.3.3 – 1.3.5]

**Table 5.7.3.4 Load Modifiers**

Modifier	Strength	Service	Fatigue
Ductility $\eta_D$	1.0	1.0	1.0
Redundancy $\eta_R$	1.0	1.0	1.0
Importance $\eta_I$	1.0	n/a	n/a
$\eta = \eta_D \cdot \eta_R \cdot \eta_I$	1.0	1.0	1.0



**D. Shear Forces  
and Bending  
Moments**

Four load combinations will be considered: Strength I, Service I, Service III, and Fatigue. As a result of the simple span configuration, only maximum  $\gamma_p$  values need to be considered.

Load effects related to settlement, thermal effects, water load, or stream pressure will not be considered.

**[3.6.2]**

Dynamic load allowance IM = 33%

$$\text{Beam Selfweight} = (704/144) \cdot (0.155 \text{ k/ft}^3) = 0.758 \text{ k/ft}$$

$$\text{Stool Weight} = (2.83 \text{ ft}) \cdot (0.208 \text{ ft}) \cdot (0.150 \text{ k/ft}^3) = 0.088 \text{ k/ft}$$

$$\text{Deck Weight} = (6.83 \text{ ft}) \cdot (0.75 \text{ ft}) \cdot (0.150 \text{ k/ft}^3) = 0.769 \text{ k/ft}$$

$$\text{Future Wearing Surface} = (0.020 \text{ k/ft}^2) \cdot (36 \text{ ft}) \cdot (1/6) = 0.120 \text{ k/ft}$$

$$\text{Barrier Weight} = 2 \cdot (0.496 \text{ k/ft}) \cdot (1/6) = 0.165 \text{ k/ft}$$

The load due to the intermediate diaphragms is calculated by referring to standard detail B403. For 40MH beams, the diaphragm consists of a steel C12 x 20.7 that is connected to the beams with 1.0' x 1.0' bent plates.

$$\text{Diaphragm Weight} \cong (6.83 \text{ ft}) \cdot (0.0207 \text{ k/ft})$$

$$+ 2 \cdot (1.0 \text{ ft}) \cdot (1.0 \text{ ft}) \cdot \left( \frac{0.375 \text{ in}}{12 \text{ in/ft}} \right) \cdot (0.490 \text{ k/ft}^3) = 0.172 \text{ kips}$$

Critical locations along the beam need to be analyzed for moments and shear. These critical locations include: the inside face of bearing, prestress transfer points, critical shear point, and tenth points along the length of the beam. These locations, dimensioned from the beam centerline of bearing, are determined as follows:

Bearing Face (inside face of bearing is the point where a crack could start at the bottom of the beam, which is the inside edge of the sole plate)

$$= X_{\text{brgface}} = \frac{L_{\text{soleplate}}}{2} = 7.5 \text{ in} = 0.63 \text{ ft}$$

Initial Transfer Point

$$= X_{\text{transfer1}} = 60 \cdot d_b - \frac{L_{\text{soleplate}}}{2} = (60 \cdot 0.6) - \frac{15}{2} = 28.5 \text{ in} = 2.38 \text{ ft}$$

Bottom Flange Debonding Transfer Points (debonding limitations and the general guidance that helped establish these locations are discussed later in this example)

$$= X_{\text{transfer2}} = X_{\text{transfer1}} + 14 = 16.4 \text{ ft}$$

$$= X_{\text{transfer3}} = X_{\text{transfer1}} + 18 = 20.4 \text{ ft}$$

$$= X_{\text{transfer4}} = X_{\text{transfer1}} + 22 = 24.4 \text{ ft}$$

Critical Shear Point (located at  $d_v$  from the inside face of bearing, calculations are shown in "F. Design Reinforcement for Shear")

$$= X_{v_{crit}} = 4.19 \text{ ft}$$

Tenth points are simply 0.1L, 0.2L, 0.3L, 0.4L, and 0.5L

The bending moments and shears for the dead and live loads were obtained with a line girder model of the bridge. They are summarized in Tables 5.7.3.5 and 5.7.3.6.

**Table 5.7.3.5  
Shear Force Summary (kips/beam)**

Load Type/Combination	Brg CL (0.0')	Brg Face (0.63')	Trans Point #1 (2.38')	Critical Shear Point (4.2')	0.1 Span Point (11.8')	Trans Point #2 (16.4')	Trans Point #3 (20.4')	0.2 Span Point (23.6')	Trans Point #4 (24.4')	0.3 Span Point (35.4')	0.4 Span Point (47.2')	0.5 Span Point (59.0')	
Dead Loads	Selfweight	45	44	43	42	36	32	29	27	26	18	9	0
	Stool	5	5	5	5	4	4	3	3	3	2	1	0
	Deck	45	45	44	42	36	33	30	27	27	18	9	0
	FWS	7	7	7	7	6	5	5	4	4	3	1	0
	Barrier	10	10	9	9	8	7	6	6	6	4	2	0
	Diaphragms	0	0	0	0	0	0	0	0	0	0	0	0
	Total	112	111	108	105	90	81	73	67	66	45	22	0
Live Loads <sup>①</sup>	Uniform Lane	28	27	27	26	22	20	19	18	17	14	10	7
	Tandem + IM	48	48	47	46	43	41	39	38	38	33	28	23
	Truck + IM	64	64	63	62	57	55	52	50	50	43	36	29
	Governing LL (Truck + IM) + Lane	92	91	90	88	79	75	71	68	67	57	46	36
Strength I Load Comb (1.25·DL+1.75·LL)	301	298	293	285	251	233	216	203	200	156	108	63	
Service I Load Comb (1.00·DL+1.00·LL)	204	202	198	193	169	156	144	135	133	102	68	36	
Service III Load Comb (1.00·DL+0.80·LL)	186	184	180	175	153	141	130	121	120	91	59	29	

① All live loads include the interior beam live load distribution factors of 0.731 and IM of 0.33.

**Table 5.7.3.6**  
**Bending Moment Summary (kip-ft/beam)**

Load Type/Combination		Brg CL (0.0')	Brg Face (0.63')	Trans Point #1 (2.38')	Critical Shear Point (4.2')	0.1 Span Point (11.8')	Trans Point #2 (16.4')	Trans Point #3 (20.4')	0.2 Span Point (23.6')	Trans Point #4 (24.4')	0.3 Span Point (35.4')	0.4 Span Point (47.2')	0.5 Span Point (59.0')	
Dead Loads	DC1	Selfweight	0	28	104 <sup>②</sup>	181	475	632	755	844	865 <sup>③</sup>	1108	1267	1319
		Stool	0	3	12	21	55	73	88	98	100	129	147	153
		Deck	0	28	106	184	482	641	766	857	878	1124	1285	1338
		Diaph.	0	0	0	1	2	3	4	4	4	6	7	7
		Total DC1	0	59	222	387	1014	1349	1613	1803	1847	2367	2706	2817
	DC2	Barrier	0	6	23	39	103	137	164	184	188	241	276	287
		FWS	0	4	16	29	75	100	119	134	137	175	201	209
		Total DC2	0	10	39	68	178	237	283	318	325	416	477	496
	Total (DC1+DC2)		0	69	261	455	1192	1586	1896	2121	2172	2783	3183	3313
	Live Loads <sup>①</sup>	Uniform Lane	0	13	47	82	216	287	343	384	393	503	575	599
Tandem + IM		0	22	82	142	373	495	591	661	677	865	985	1020	
Truck + IM		0	29	110	192	499	661	786	877	897	1132	1283	1319	
Governing LL (Truck+IM) +Lane		0	42	157	274	715	948	1129	1261	1290	1635	1858	1918 <sup>④</sup>	
Strength I Load Comb (1.25·DL+1.75·LL)		0	160	601	1048	2741	3642	4346	4858	4973	6340	7230	7498	
Service I Load Comb (1.00·DL+1.00·LL)		0	111	418	729	1907	2534	3025	3382	3462	4418	5041	5231	
Service III Load Comb (1.00·DL+0.80·LL)		0	103	387	674	1764	2344	2799	3130	3204	4091	4669	4847	
Fatigue I Load Comb (1.75·LL)		-	-	-	-	-	-	-	-	-	-	-	1008	

① All live loads include the interior beam live load distribution factor of 0.538 and IM of 0.33.

② Beam selfweight at strand release = 132 k-ft (beam in casting bed with span length equal to overall beam length of 119.25)

Beam selfweight at erection on bearings = 104 k-ft (beam span length equal to design span of 118.0 ft)

③ Beam selfweight at strand release = 893 k-ft (beam in casting bed with span length equal to overall beam length of 119.25 ft)

Beam selfweight at erection on bearings = 865 k-ft (beam span length equal to design span of 118.0 ft)

④ Fatigue live load = 576 k-ft (includes interior beam live load distribution factor of 0.315 and IM of 0.15 applied to fatigue truck only)

**E. Design Beam  
Pretensioning With  
Debonded Strands  
for Control of End  
Stresses**

Typically, the tension at the bottom of the beam at midspan in its final configuration after all losses have occurred dictates the required level of prestressing.

**1. Estimate Required Prestress**

Use the Service III load combination

Bottom of beam stress:

$$f_{\text{serv3bot}} = \left( \frac{M_{\text{DC1}}}{S_{\text{gb}}} \right) + \left( \frac{M_{\text{DC2}}}{S_{\text{cb}}} \right) + \left( \frac{M_{\text{LL}} \cdot 0.8}{S_{\text{cb}}} \right)$$

$$= \left( \frac{2817 \cdot 12}{8,246} \right) + \left( \frac{496 \cdot 12}{12,917} \right) + \left( \frac{1918 \cdot 12 \cdot 0.8}{12,917} \right) = 5.99 \text{ ksi}$$

For 300 ksi strands, MnDOT practice is to jack to an initial prestress force of  $0.72f_{\text{pu}}$ . As a starting point, the total prestress losses will be assumed to be 24%. This results in an effective prestress of

$$f_{\text{pe}} = 0.72 \cdot f_{\text{pu}} \cdot (1 - 0.24) = 0.72 \cdot 300 \cdot 0.76 = 164.2 \text{ ksi}$$

Strands are typically placed on a 2" grid. Referring to BDM Figures 5.4.6.2 to determine a starting point for the number of strands, assume 50 strands and choose a pattern that provides the greatest eccentricity for the prestressing force. This pattern will fill all the straight strand locations in the bottom flange. The centroid of this 50 strand pattern is:

$$y_{\text{str}} = \left[ \frac{\sum (\# \text{ of strands}) \cdot (y \text{ of strands})}{(\text{total } \# \text{ of strands})} \right]$$

$$= \left[ \frac{18 \cdot (2 + 4) + (10 \cdot 6) + (4 \cdot 8)}{50} \right] = 4.00 \text{ in}$$

Using the centroid of this group as an estimate of the strand pattern eccentricity results in

$$e_{50} = y_g - y_{\text{str}} = 18.07 - 4.00 = 14.07$$

The area,  $A_{\text{strand}}$ , of a 0.6" diameter 7-wire strand is  $0.217 \text{ in}^2$

The axial compression produced by the prestressing strands is

$$P = A_s \cdot f_{pe} = n_{\text{strands}} \cdot A_{\text{strand}} \cdot f_{pe}$$

The internal moment produced by the prestressing strands is

$$M_{p/s} = A_s \cdot f_{pe} \cdot e_{50} = n_{\text{strands}} \cdot A_{\text{strand}} \cdot f_{pe} \cdot e_{50}$$

The allowable tension after losses =  $0.19 \cdot \sqrt{f'_c} = 0.19 \cdot \sqrt{9.5} = 0.59$  ksi

This moment and axial compression from the prestress,  $f_{pscomp}$ , must reduce the bottom flange tension from 5.99 ksi tension to the allowable tension of 0.59 ksi.

$$f_{pscomp} = 5.99 - 0.59 = 5.40 \text{ ksi}$$

Knowing that

$$f_{pscomp} = \frac{P}{A} + \frac{M_{p/s}}{S_b}$$

and substituting and solving for  $n_{\text{strands}}$ , we get an estimate for the required number of strands:

$$\begin{aligned} n_{\text{strands}} &= \frac{f_{pscomp}}{A_{\text{strand}} \cdot f_{pe} \cdot \left( \frac{1}{A} + \frac{e_{50}}{S_b} \right)} = \frac{5.40}{0.217 \cdot 164.2 \cdot \left( \frac{1}{704} + \frac{14.07}{8246} \right)} \\ &= 48.5 \text{ strands} \end{aligned}$$

Try a strand pattern with 48 strands to begin.

After reviewing Bridge Details Part II Figure 5-397.503, the trial strand pattern shown in Figure 5.7.3.4 was selected.

The properties of this strand pattern at midspan are:

$$y_{\text{strand}} = \left[ \frac{18 \cdot (2 + 4) + (10 \cdot 6) + (2 \cdot 8)}{48} \right] = 3.83 \text{ in}$$

$$e_{\text{strand}} = y_b - y_{\text{strand}} = 18.07 - 3.83 = 14.24 \text{ in}$$

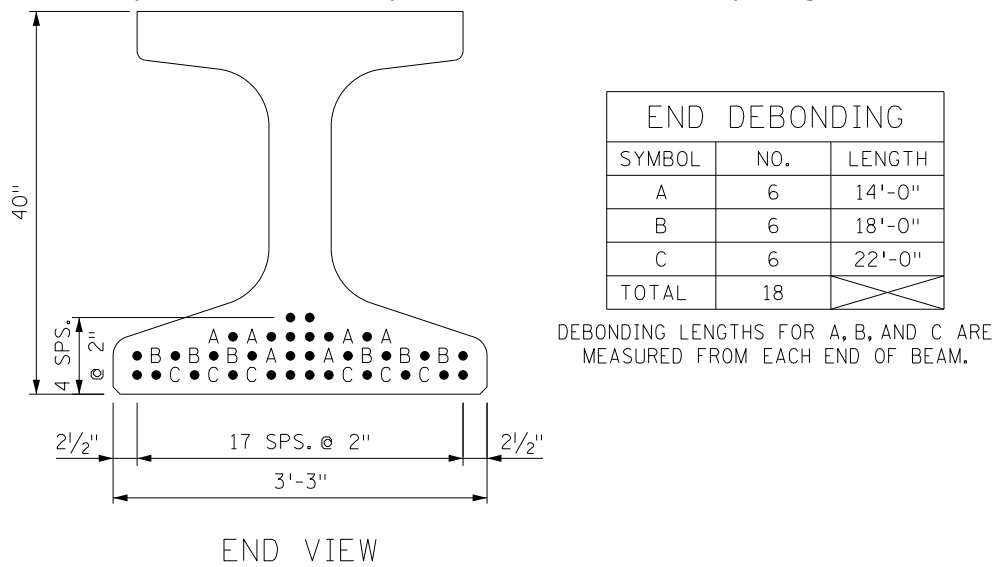
[5.9.4.3.3]

**2. Debonding**

Typically, tensile stress on the top flange of the beam near its ends dictate the amount of debonding. This example uses the maximum amount of debonding given the selected 48 strands. Debonding limitations are as follows:

- Maximum number of strands that terminate debonding at any given location:
  - 4 – When ten or fewer total strands are debonded.
  - 6 – When eleven or more total strands are debonded.
- Maximum of 45% of strands debonded in each row.
- Fully bonded strand location requirements:
  - MnDOT shape base strands (refer to Figure 5.4.3.1).
  - Bottom flange strands within the horizontal limits of web.
  - Outermost strands within the full-width section of the bottom flange.
- Alternate bonded and debonded strand locations both horizontally and vertically.
- Longitudinal spacing of debonding termination points shall be at least  $60 d_b$  (3 ft).
- Debond strands symmetrically about beam centerline.
- For simple span precast, pretensioned girders, limit debonding lengths from the end of the beam to 20% of the total span length (23.6 feet).

After analyzing the section, the following layout which maximizes debonding, 18 debonded strands, was obtained. The initial termination point of 14 feet was chosen arbitrarily and will be confirmed later. Subsequent terminations points were chosen at a spacing of 4 ft.



**Figure 5.7.3.4**

Certain critical sections need to be tested. Depending on the stress checks at transfer, consider altering the number of debonded strands, debonding termination points, or add strands in the top flange. Before testing these critical locations, the losses due to prestressing need to be calculated.

**[5.9.3]****3. Prestress Losses**

Prestress losses are computed using the approximate method.

**[5.9.3.2.3]****Elastic Shortening Loss**

Use the alternative equation presented in the LRFD C5.9.3.2.3a.

$$\Delta f_{pES} = \frac{A_{ps} \cdot f_{pbt} \cdot (I_g + e_m^2 \cdot A_g) - e_m \cdot M_g \cdot A_g}{A_{ps} \cdot (I_g + e_m^2 \cdot A_g) + \frac{A_g \cdot I_g \cdot E_{ci}}{E_p}}$$

$$A_{ps} = (\# \text{ of strands}) \cdot (\text{strand area}) = 48 \cdot 0.217 = 10.42 \text{ in}^2 \quad (\text{midspan})$$

$$f_{pbt} = f_{pj} = 216.0 \text{ ksi}$$

$$e_m = e_{\text{strandmid}} = 14.24 \text{ in}$$

$$\frac{A_g \cdot I_g \cdot E_{ci}}{E_p} = \frac{704 \cdot (149,002) \cdot (4578)}{28,500} = 16,849,836 \text{ in}^6$$

$$A_{ps} \cdot (I_g + e_m^2 \cdot A_g) = 10.42 [149,002 + (14.24)^2 \cdot (704)] = 3,040,112 \text{ in}^6$$

$$\Delta f_{pES} = \frac{216.0 \cdot (3,040,112) - 14.24 \cdot (1319) \cdot (12) \cdot (704)}{3,040,112 + 16,849,836} = 25.0 \text{ ksi}$$

**[5.9.3.3]****Long Term Losses**

Use the approximate equation in the LRFD 5.9.3.3

$$\Delta f_{pLT} = 10.0 \cdot \frac{f_{pj} \cdot A_{ps}}{A_g} \cdot \gamma_h \cdot \gamma_{st} + 12 \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR}$$

For an average humidity in Minnesota of 73%

$$\gamma_h = 1.7 - 0.01 \cdot H = 1.7 - 0.01 \cdot 73 = 0.97$$

$$\gamma_{st} = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 8.0} = 0.56$$

For low relaxation strand,  $\Delta f_{pR} = 2.4$

$$\begin{aligned} \Delta f_{pLT} &= 10.0 \cdot \frac{216.0 \cdot (10.42)}{704} \cdot 0.97(0.56) + 12.0(0.97)(0.56) + 2.4 \\ &= 26.3 \text{ ksi} \end{aligned}$$

**[5.9.3.1]****Total Losses**

$$\Delta f_{pt} = \Delta f_{pES} + \Delta f_{pLT} = 25.0 + 26.3 = 51.3 \text{ ksi}$$

$$f_{pe} = f_{pj} - \Delta f_{pt} = 216.0 - 51.3 = 164.7 \text{ ksi}$$

$$\text{prestress loss percentage} = \frac{\Delta f_{pt}}{f_{pj}} \cdot 100 = \frac{51.3}{216.0} \cdot 100 = 23.8 \%$$

Jacking force:

$$P_{jack} = A_{ps} \cdot (f_{pj}) = 10.42 \cdot (216.0) = 2251 \text{ kips}$$

Initial prestress force after transfer through midspan:

$$P_i = A_{ps} \cdot (f_{pj} - \Delta f_{pES}) = 10.42 \cdot (216.0 - 25.0) = 1990 \text{ kips}$$

Prestress force after all losses through midspan:

$$P_e = A_{ps} \cdot f_{pe} = 10.42 \cdot 164.7 = 1716 \text{ kips}$$

Prestress Forces for Transfer Point #1

$$A_{ps} = (\# \text{ of strands}) \cdot (\text{strand area}) = 30 \cdot 0.217 = 6.51 \text{ in}^2$$

Initial prestress force after transfer at Transfer Point #1

$$P_i = A_{ps} \cdot (f_{pj} - \Delta f_{pES}) = 6.51 \cdot (216.0 - 25.0) = 1243 \text{ kips}$$

Prestress force after all losses at Transfer Point #1

$$P_e = A_{ps} \cdot f_{pe} = 6.51 \cdot 164.7 = 1072 \text{ kips}$$

**[5.9.2.3.1]**

**4. Stresses at Transfer (compression +, tension -)  
Stress Limits for P/S Concrete at Release**

This example checks the top stress at Transfer Point #1 for fully bonded strands ( $x=2.38'$ ) and the bottom compression stress at Transfer Point #4 for strands that terminate at 22 feet ( $x=24.4'$ ). These checks will help to determine if the amount of debonding is sufficient. Consider reducing the amount or length of debonding if compression stresses at transfer points are significantly passing. Consider increasing the amount of debonding (within allowable limits) or adding top flange strands if tension stresses at transfer points are failing. Only one tension and compression check at release are shown for brevity. As with all limit states checked in this example, additional locations are often required based on strand and debonding layout.



Compression in the concrete is limited to:

$$f_{climrel} = 0.65 \cdot f'_{ci} = 0.65 \cdot 8.0 = 5.20 \text{ ksi}$$

For tension, MnDOT uses the AASHTO Table 5.9.2.3.1b-1 stress limits for "areas with bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force" for beam design. The tension in the concrete is limited to:

$$f_{tlimrel} = -0.24 \cdot \sqrt{f'_{ci}} = -0.24 \cdot \sqrt{8.0} = -0.68 \text{ ksi}$$

### Confirm Bonded Reinforcement Tension Limit

Using the method presented in the LRFD C5.9.2.31b, confirm the required amount of bonded reinforcement is present in the top flange to resist the calculated tensile force. The total tensile force is determined by integrating the stress over the entire tensile zone. This can be estimated by summing the tensile force from discrete subzones using simplified beam geometry.

Centroid of strand pattern at Transfer Point #1:

$$y_{strand} = \left[ \frac{(12 \cdot 2) + (10 \cdot 4) + (6 \cdot 6) + (2 \cdot 8)}{30} \right] = 3.87 \text{ in}$$

The eccentricity of the strand pattern at transfer point #1 is:

$$e_{strand} = y_b - y_{strand} = 18.07 - 3.87 = 14.20 \text{ in}$$

At this point, the beam is sitting in the casting bed. The beam will camber upward when the strands are released, so the span length used to determine the selfweight moment is the end-to-end beam length of 119.25 feet.

The internal prestress moment at Transfer Point #1 is:

$$P_i \cdot e_{strand} = 1243 \cdot 14.20 = 17,651 \text{ kip-in}$$

$$\begin{aligned} \text{Top stress due to P/S} &= \left( \frac{P_i}{A_g} \right) - \left( \frac{P_i \cdot e_{strand}}{S_{gt}} \right) = \left( \frac{1243}{704} \right) - \left( \frac{17,651}{6794} \right) \\ &= -0.83 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{Bottom stress due to P/S} &= \left( \frac{P_i}{A_g} \right) + \left( \frac{P_i \cdot e_{strand}}{S_{gb}} \right) = \left( \frac{1243}{704} \right) + \left( \frac{17,651}{8246} \right) \\ &= 3.91 \text{ ksi} \end{aligned}$$

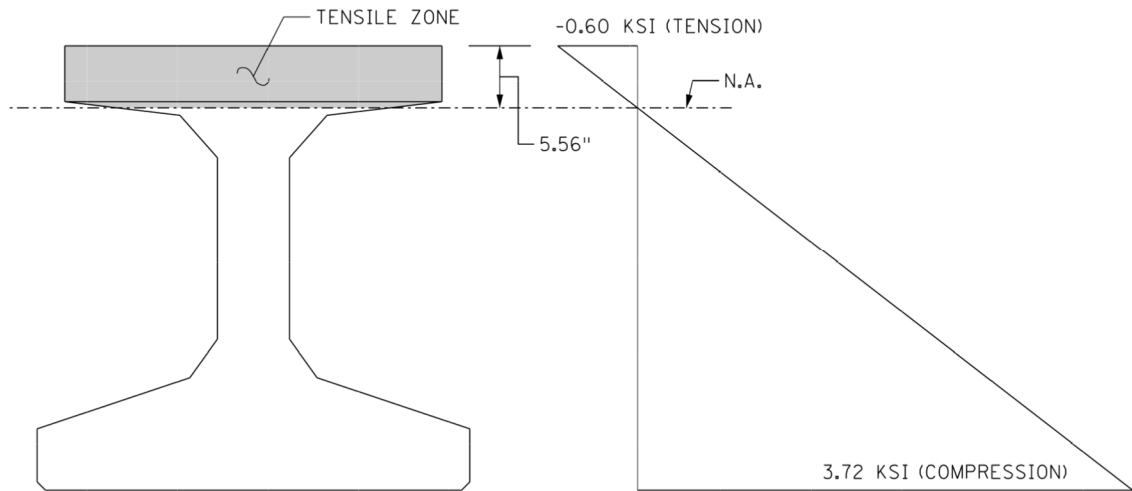
$$\text{Top stress due to selfweight} = \left( \frac{M_{swtr}}{S_{gt}} \right) = \left( \frac{132 \cdot 12}{6794} \right) = 0.23 \text{ ksi}$$

$$\text{Bottom stress due to selfweight} = -\left(\frac{M_{\text{swtr}}}{S_{\text{gb}}}\right) = -\left(\frac{132 \cdot 12}{8246}\right) = -0.19 \text{ ksi}$$

$$\text{Top stress} = -0.83 + 0.23 = -0.60 \text{ ksi}$$

$$\text{Bottom stress} = 3.91 - 0.19 = 3.72 \text{ ksi}$$

$$\text{Depth of Neutral Axis} = \frac{0.60}{\left(\frac{0.60 + 3.72}{40}\right)} = 5.56 \text{ in}$$



**Figure 5.7.3.5**

Using the stresses distribution and simplified beam geometry shown in Figure 5.7.3.5, the total tensile force at Transfer Point #1 is calculated in Table 5.7.3.7 and subsequent equations.

**Table 5.7.3.7**  
**Summation of Tension Force**

Slice	y <sub>top</sub> (in)	y <sub>bot</sub> (in)	y <sub>cg</sub> (in)	A <sub>i</sub> (in <sup>2</sup> )	f <sub>i</sub> (ksi)	T <sub>i</sub> (kips)
1	0.00	5.00	2.50	170.0	-0.33	56.1
2	5.00	5.56	5.26	16.3	-0.03	0.5

$$T = \sum f_i \cdot A_i = \sum T_i = 56.1 + 0.5 = 56.6 \text{ kips}$$

$$f_s = 0.5 \cdot f_y = 0.5 \cdot 60 = 30.0 \text{ ksi} \leq 30.0 \text{ ksi}$$

$$A_{s\_req} = \frac{T}{f_s} = \frac{56.6}{30.0} = 1.89 \text{ in}^2$$

The above calculation shows 1.89 in<sup>2</sup> of developed reinforcement is required in the top flange to accommodate the release tension stress at Transfer Point #1. The standard amount of top flange reinforcement for a 40MH beam per Bridge Details Part II Figure 5-397.503 is four #8 bars (G806E). The development of the G806E bars can be calculated as follows:

**[5.10.8.2.1]**

$$l_{db} = 2.4 \cdot d_b \cdot \frac{f_y}{\sqrt{f'_c}} = 2.4 \cdot 1.0 \cdot \frac{60}{\sqrt{8}} = 50.9 \text{ in}$$

The following modification factors are assumed:

$$\lambda = 1.0$$

$$\lambda_{rl} = 1.0$$

$$\lambda_{cf} = 1.5$$

$$\lambda_{er} = 1.0$$

$$\lambda_{rc} = \frac{d_b}{c_b + k_{tr}} = \frac{1.0}{2.38 + 0} = \quad (\text{conservatively assumes } k_{tr} = 0)$$

$$= 0.42 \quad (0.4 \leq \lambda_{rc} \leq 1.0)$$

$$l_d = l_{db} \cdot \left( \frac{\lambda_{rl} \cdot \lambda_{cf} \cdot \lambda_{rc} \cdot \lambda_{er}}{\lambda} \right) = 50.9 \cdot \left( \frac{1.0 \cdot 1.5 \cdot 0.42 \cdot 1.0}{1.0} \right) = 32.1 \text{ in}$$

Assuming a 1½ inch clear cover to the end of beam, this calculation shows the G806E bars are fully developed at Transfer Point #1. Although not quantified in this example, the top leg of two G505E bars also contribute to the bonded reinforcement area as the G806E bar develops at the end of beam.

$$A_{s\_prov} = 4 \cdot 0.79 = 3.16 \text{ in}^2 \geq A_{s\_req} = 1.89 \text{ in}^2 \quad \text{OK}$$

Because the provided area of developed reinforcement (3.16 in<sup>2</sup>) is larger than the required area (1.89 in<sup>2</sup>), the previously calculated full tension stress limit ( $f_{tlimrel} = -0.68$  ksi) can be used. Alternatively, BDM Table 5.4.2.1 can be used when applicable, in lieu of the process shown in LRFD C5.9.2.31b, to confirm the required amount of developed reinforcement is present in the top flange for standard MnDOT beam shapes.

**Check Release Stresses at Transfer Points #1 and #4**

Top stress at Transfer Point #1 was calculated in the previous section as follows:

$$\text{Top stress at Transfer Point \#1} = -0.60 \text{ ksi}$$

$$f_{tlimrel} = -0.68 \text{ ksi} \quad \text{OK}$$

Bottom stress at Transfer Point #4 can be calculated as follows:

$$P_i \cdot e_{\text{strand}} = 1990 \cdot 14.24 = 28,338 \text{ kip-in}$$

$$\begin{aligned} \text{Bottom stress due to P/S} &= \left( \frac{P_i}{A_g} \right) + \left( \frac{P_i \cdot e_{\text{strand}}}{S_{gb}} \right) = \left( \frac{1990}{704} \right) + \left( \frac{28,338}{8246} \right) \\ &= 6.26 \text{ ksi} \end{aligned}$$

$$\text{Bottom stress due to selfweight} = - \left( \frac{M_{\text{swtr}}}{S_{gb}} \right) = - \left( \frac{893 \cdot 12}{8246} \right) = -1.30 \text{ ksi}$$

Bottom stress at Transfer Point #4 = 6.26 – 1.30 = 4.96 ksi < 5.20 ksi OK

If the tension stress at Transfer Point #1 or compressive stress at Transfer Point #4 fail, consider the following options to rectify the issue:

- Increase initial concrete compressive strength,  $f'_{ci}$  (up to 8 ksi)
- Increase the amount or length of debonding used
- Raise the center of gravity of prestressing strands
- Add permanent or temporary top flange strands.

The initial concrete strength,  $f'_{ci}$ , was assumed to be 8.0 ksi. For the most economical beam, the designer should choose the lowest required  $f'_{ci}$  for the beam. This can be determined by substituting the calculated maximum compression and tension in the stress limit equations and solving for  $f'_{ci}$ .

$$\text{Lowest required } f'_{\text{ci req comp}} = \frac{f_{\text{compTP\#4}}}{0.65} = \left( \frac{4.96}{0.65} \right) = 7.63 \text{ ksi}$$

$$\text{Lowest required } f'_{\text{ci req ten}} = \left( \frac{f_{\text{tenTP\#1}}}{0.24} \right)^2 = \left( \frac{-0.60}{0.24} \right)^2 = 6.25 \text{ ksi}$$

Controlling initial concrete strength  $f'_{ci} = 7.7 \text{ ksi}$

When updating the initial concrete strength, prestress losses must be recalculated. The reduced initial concrete strength of 7.7 ksi increases the prestress losses in the final condition. This loss of prestress force necessitates a final concrete strength,  $f'_c$ , of greater than 9.5 ksi, the maximum allowable. Due to this, the initial concrete strength will remain at 8.0 ksi despite release stress checks passing at a lower concrete strength.

Proceed to the service and fatigue stress checks after final losses.

[5.9.2.3.2]

**5. Stresses at Service Loads (compression +, tension -)****Stress Limits for P/S Concrete after All Losses**

Compression in the concrete is limited to (Service I Load Combination):

$$f_{climf1} = 0.45 \cdot f'_c = 0.45 \cdot 9.5 = 4.28 \text{ ksi}$$

(for prestress and permanent loads)

Check the bottom stress at transfer points and the top stress at midspan against this limit.

$$f_{climf2} = 0.60 \cdot \phi_w \cdot f'_c = 0.60 \cdot 1.0 \cdot 9.5 = 5.70 \text{ ksi}$$

(for live load, prestress, permanent loads, and transient loads)

Check the top stress at midspan against this limit.

[5.5.3.1]

Compression in concrete is limited to (Fatigue I Load Combination):

$$f_{climfat} = 0.40 \cdot f'_c = 0.40 \cdot 9.5 = 3.80 \text{ ksi}$$

(for live load and  $1/2$  of prestress and permanent loads)

Check the top stress at midspan against this limit.

Tension in the concrete is limited to (Service III Load Combination):

$$f_{flimf} = -0.19 \cdot \sqrt{f'_c} = -0.19 \cdot \sqrt{9.5} = -0.586 \text{ ksi}$$

Check the bottom stress at midspan against this limit.

**Check Stresses at Midspan After Losses:**

Bottom stress

$$\begin{aligned} &= -\left(\frac{M_{DC1}}{S_{gb}}\right) - \left(\frac{M_{DC2}}{S_{cb}}\right) - \left(\frac{M_{LL} \cdot 0.8}{S_{cb}}\right) + \left(\frac{P_e}{A_g}\right) + \left(\frac{P_e \cdot e_{strand}}{S_{gb}}\right) \\ &= -\left(\frac{2817 \cdot 12}{8246}\right) - \left(\frac{496 \cdot 12}{12,917}\right) - \left(\frac{1918 \cdot 12 \cdot 0.8}{12,917}\right) + \left(\frac{1716}{704}\right) + \left(\frac{1716 \cdot 14.24}{8246}\right) \\ &= -0.585 \text{ ksi} < -0.586 \text{ ksi} \quad \text{OK} \end{aligned}$$

Top stress due to all loads

$$\begin{aligned} &= \left(\frac{P_e}{A_g}\right) - \left(\frac{P_e \cdot e_{strand}}{S_{gt}}\right) + \left(\frac{M_{DC1}}{S_{gt}}\right) + \left(\frac{M_{DC2} + M_{LL}}{S_{gtc}}\right) \\ &= \left(\frac{1716}{704}\right) - \left(\frac{1716 \cdot 14.24}{6794}\right) + \left(\frac{2817 \cdot 12}{6794}\right) + \left[\frac{(496 + 1918) \cdot 12}{42,761}\right] \\ &= 4.49 \text{ ksi} < 5.70 \text{ ksi} \quad \text{OK} \end{aligned}$$

Top stress due to permanent loads

$$= \left(\frac{P_e}{A_g}\right) - \left(\frac{P_e \cdot e_{strand}}{S_{gt}}\right) + \left(\frac{M_{DC1}}{S_{gt}}\right) + \left(\frac{M_{DC2}}{S_{gtc}}\right)$$

$$\begin{aligned}
 &= \left( \frac{1716}{704} \right) - \left( \frac{1716 \cdot 14.24}{6794} \right) + \left( \frac{2817 \cdot 12}{6794} \right) + \left( \frac{496 \cdot 12}{42,761} \right) \\
 &= 3.96 \text{ ksi} < 4.28 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

Top stress due to fatigue live load plus ½ the sum of prestress and permanent loads

$$\begin{aligned}
 &= \frac{1}{2} \left( \left( \frac{P_e}{A_g} \right) - \left( \frac{P_e \cdot e_{\text{strand}}}{S_{gt}} \right) + \left( \frac{M_{DC1}}{S_{gt}} \right) + \left( \frac{M_{DC2}}{S_{gtc}} \right) \right) + \left( \frac{M_{LL}}{S_{gtc}} \right) \\
 &\frac{1}{2} \left( \left( \frac{1716}{704} \right) - \left( \frac{1716 \cdot 14.24}{6794} \right) + \left( \frac{2817 \cdot 12}{6794} \right) + \left( \frac{496 \cdot 12}{42,761} \right) \right) + \left( \frac{1008 \cdot 12}{42,761} \right) \\
 &= 2.26 \text{ ksi} < 3.80 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

### Check the Compression Stresses at Transfer Points After Losses

Bottom flange stress at Transfer Point # 1 due to prestress and permanent loads

$$\begin{aligned}
 &= \frac{P_e}{A_g} + \frac{P_e \cdot e_{\text{strand}}}{S_{gb}} - \left( \frac{M_{DC1}}{S_{gb}} \right) - \left( \frac{M_{DC2}}{S_{gbc}} \right) \\
 &= \frac{1072}{704} + \frac{1072 \cdot 14.20}{8246} - \left( \frac{222 \cdot 12}{8246} \right) - \left( \frac{39 \cdot 12}{12,917} \right) \\
 &= 3.01 \text{ ksi} < 4.28 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

Bottom flange stress at Transfer Point # 4 due to prestress and permanent loads

$$\begin{aligned}
 &= \frac{P_e}{A_g} + \frac{P_e \cdot e_{\text{strand}}}{S_{gb}} - \left( \frac{M_{DC1}}{S_{gb}} \right) - \left( \frac{M_{DC2}}{S_{gbc}} \right) \\
 &= \frac{1716}{704} + \frac{1716 \cdot 14.24}{8246} - \left( \frac{1847 \cdot 12}{8246} \right) - \left( \frac{325 \cdot 12}{12,917} \right) \\
 &= 2.41 \text{ ksi} < 4.28 \text{ ksi} \quad \text{OK}
 \end{aligned}$$

As discussed in the release stress calculations, the highest tensile stress at midpoint after losses is the most critical location (0.585 ksi < 0.586 ksi). This calculation confirms reducing initial concrete strength, thus increasing prestress losses, is not possible in the final condition. This closes the loop as to why the initial concrete strength will remain at 8.0 ksi.

The final concrete strength,  $f'_c$ , was assumed to be 9.5 ksi. For the most economical beam, the designer should choose the lowest required  $f'_c$  for

the beam. This is determined by substituting the calculated maximum tensile stress for  $f_{\text{limf}}$  in the tension limit equation and solving for  $f'_c$ .

$$\text{Lowest required } f'_c = \left( \frac{f_{\text{limf}}}{0.19} \right)^2 = \left( \frac{-0.585}{0.19} \right)^2 = 9.48 \text{ ksi}$$

The assumed concrete strength cannot be reduced.

Keep  $f'_c = 9.5$  ksi

#### [5.5.4]

### 6. Flexure – Strength Limit State

Resistance factors at the strength limit state are:

$$\phi = 1.00 \text{ for flexure and tension (assumed)}$$

$$\phi = 0.90 \text{ for shear and torsion}$$

$$\phi = 1.00 \text{ for tension in steel in anchorage zones}$$

Strength I design moment,  $M_u$ , is 7498 kip-ft at midspan.

From previous calculations, distance to strand centroid from bottom of the beam at midspan is:

$$y_{\text{strand}} = 3.83 \text{ in}$$

Similar to Grade 270 strands, the yield strength,  $f_{py}$  is taken as  $0.9 \cdot f_{pu}$ .

$$f_{py} = 0.9 \cdot f_{pu} = 0.9 \cdot 300 = 270 \text{ ksi}$$

#### [5.6.3.1.1]

$$k = 2 \cdot \left( 1.04 - \frac{f_{py}}{f_{pu}} \right) = 2 \cdot \left( 1.04 - \frac{270}{300} \right) = 0.28$$

$$\begin{aligned} d_p &= (\text{beam height}) + \text{stool} + \text{deck} - y_{\text{strand}} \\ &= 40 + 1.5 + 8.5 - 3.83 = 46.17 \text{ in} \end{aligned}$$

Begin by assuming the neutral axis lies in the deck.

For  $f'_c = 4.0$  ksi,  $\beta_1 = 0.85$  and  $\alpha_1 = 0.85$ .

Then

$$\begin{aligned} c &= \frac{A_{ps} \cdot f_{pu}}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_e + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}} \\ &= \frac{10.42 \cdot 300}{0.85 \cdot 4.0 \cdot 0.85 \cdot 82 + 0.28 \cdot 10.42 \cdot \left( \frac{300}{46.17} \right)} = 12.21 \text{ in} \end{aligned}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 12.21 = 10.38 \text{ in}$$

Compression block depth is greater than the thickness of the slab (8.5 in), so T-section behavior must be considered. The "web width",  $b_w$ , of the T-section is the beam flange width, which is 34 in.

Then

$$c = \frac{A_{ps} \cdot f_{pu} - \alpha_1 \cdot f'_c \cdot (b - b_w) \cdot h_f}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_w + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}}$$

$$= \frac{10.42 \cdot 300 - 0.85 \cdot 4.0 \cdot (82 - 34) \cdot 8.5}{0.85 \cdot 4.0 \cdot 0.85 \cdot 34 + 0.28 \cdot 10.42 \cdot \frac{300}{46.17}} = 14.83 \text{ in}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 14.83 = 12.61 \text{ in}$$

The revised compression block depth is less than the thickness of the slab plus the flange thickness (15 in), so T-section behavior is confirmed. If the revised compression block depth had been greater than 15 inches, the section would be acting as a stepped T-section and a strain compatibility approach would have been necessary.

$$f_{ps} = f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right) = 300 \cdot \left(1 - 0.28 \cdot \frac{14.83}{46.17}\right) = 273.0 \text{ ksi}$$

The internal lever arm between compression and tension flexural force components is:

$$d_p - \frac{a}{2} = 46.17 - \frac{12.61}{2} = 39.87 \text{ in}$$

Then:

$$M_n = A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + \alpha_1 \cdot f'_c \cdot (b - b_w) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

$$= 10.42 \cdot 273.0 \cdot 39.87 + 0.85 \cdot 4.0 \cdot (82 - 34) \cdot 8.5 \cdot \left(\frac{12.61}{2} - \frac{8.5}{2}\right)$$

$$= 116,267 \text{ kip-in} = 9689 \text{ kip-ft}$$

$$M_r = \phi M_n = 1.0 \cdot 9689 = 9689 \text{ kip-ft} > M_u = 7498 \text{ kip-ft} \quad \text{OK}$$

#### [5.5.4.2]

Validate the assumption of 1.0 for the resistance factor:

Concrete compression strain limit  $\epsilon_c = 0.003$

Reinforcement tension-controlled strain limit  $\epsilon_{tl} = 0.005$

Distance to extreme tension strand  $d_t = 40 + 1.5 + 8.5 - 2 = 48$  in

Referring to LRFD Figure C5.6.2.1-1 and using similar triangles in the prestressing strand,  $\epsilon_t$ :

$$\epsilon_t = (d_t - c) \cdot \left(\frac{\epsilon_c}{c}\right) = (48 - 14.83) \cdot \left(\frac{0.003}{14.83}\right) = 0.0067 > 0.005$$

Therefore  $\phi = 1.0$ , which matches the assumption



**[5.6.3.3]****7. Minimum Reinforcement**

To prevent brittle failure, an adequate amount of reinforcement is required. Check that the section can carry the smaller of:

- 3)  $1.33M_u$
- 4) Cracking Moment,  $M_{cr}$

At midspan,  $1.33M_u = 1.33 \cdot 7498 = 9972$  kip-ft

Lightweight concrete is not being used, so concrete density factor  $\lambda = 1.0$ .

$$f_r = 0.24 \cdot \lambda \cdot \sqrt{f'_c} = 0.24 \cdot 1.0 \cdot \sqrt{9.5} = 0.74 \text{ ksi}$$

$$\begin{aligned} f_{cpe} = f_{peb} &= \frac{P_e}{A_g} + \frac{P_e \cdot e_{strand}}{S_{gb}} \\ &= \frac{1716}{704} + \frac{1716 \cdot 14.24}{8246} = 5.40 \text{ ksi} \end{aligned}$$

$$\begin{aligned} M_{cr} &= \gamma_3 \cdot \left[ (\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}) \cdot S_{gbc} - M_{dnc} \cdot \left( \frac{S_{gbc}}{S_{gb}} - 1 \right) \right] \\ &= 1.0 \cdot \left[ (1.6 \cdot 0.74 + 1.1 \cdot 5.40) \cdot 12,917 - 2817 \cdot 12 \cdot \left( \frac{12,917}{8246} - 1 \right) \right] \\ &= 72,872 \text{ kip-in} = 6073 \text{ kip-ft} < 9972 \text{ kip-ft} \quad M_{cr} \text{ GOVERNS} \end{aligned}$$

$$M_r = \phi M_n = 9689 \text{ kip-ft} > 6073 \text{ kip-ft} \quad \text{OK}$$

**F. Design  
Reinforcement for  
Shear  
[5.7]**

**1. Vertical Shear Design  
Determine  $d_v$  and Critical Section for Shear**

Begin by determining the effective shear depth  $d_v$  at the critical section for shear. The critical location for shear  $x_{vcrit}$  is defined as the distance  $d_v$  from the internal face of support. The internal face is assumed to be at the inside edge of the 15 inch long sole plate.

The effective shear depth is taken as the greatest of:

**[5.7.2.8]**

$$d_v = d_p - \frac{a}{2} \quad \text{or} \quad 0.72h_{comp} \quad \text{or} \quad 0.9d_e$$

**[5.7.3.4.2]**

AASHTO is unclear on which strands to consider when determining  $d_p$  for calculating  $d_v$ . Considering LRFD Figure C5.7.2.8-1, it appears that  $d_v$  is based on calculating  $d_p$  and  $d_e$  for the strands found on the flexural tension side of the neutral axis. But for shear calculations, LRFD Article 5.7.3.4.2 and Figure 5.7.3.4.2-1 define  $A_{ps}$  as the strands found on the flexural tension side of  $\frac{1}{2}$  the height of the composite section. To keep computations simple, yet reasonably accurate, follow the LRFD Article 5.7.3.4.2 definition and consider only the prestressing strands found below  $\frac{1}{2}$  the height of the composite section when calculating  $d_p$ . Therefore, only bonded prestressing strands in the bottom flange are considered for debonded beams.

The flexural tension side of the member is defined as:

$$\frac{h_{\text{comp}}}{2} = \frac{50}{2} = 25 \text{ in}$$

The centroid of bonded bottom flange prestressing strands is at:

$$y_{\text{sstr}} = \left[ \frac{(12 \cdot 2) + (10 \cdot 4) + (6 \cdot 6) + (2 \cdot 8)}{30} \right]$$

$$= 3.87 \text{ in}$$

With this strand centroid,  $d_p$  can be computed for the composite section:

$$d_p = h_{\text{comp}} - y_{\text{sstr}} = 50 - 3.87 = 46.13 \text{ in}$$

Recalculate the value of the compression block depth "a" considering only the bonded prestressing strands at the critical shear section:

$$A_{ps} = A_{\text{sps}} = (\# \text{ of bonded strands}) \cdot (\text{strand area}) = 30 \cdot 0.217 = 6.51 \text{ in}^2$$

Begin by assuming the neutral axis lies in the deck.

**[5.6.3.1.1]**

$$c = \frac{A_{ps} \cdot f_{pu}}{\alpha_1 \cdot f'_c \cdot \beta_1 \cdot b_e + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}}$$

$$= \frac{6.51 \cdot 300}{0.85 \cdot 4.0 \cdot 0.85 \cdot 82 + 0.28 \cdot 6.51 \cdot \frac{300}{46.13}} = 7.85 \text{ in}$$

$$a = \beta_1 \cdot c = 0.85 \cdot 7.85 = 6.67 \text{ in}$$

Compression block depth is less than 8.5", the thickness of the slab, so T-section behavior is not considered.

$$d_v = d_p - \frac{a}{2} = 46.13 - \frac{6.67}{2} = 42.80 \text{ in}$$

But the effective shear depth  $d_v$  need not be less than

$$d_v \geq 0.72 \cdot h_{\text{comp}} = 0.72 \cdot 50 = 36.0 \text{ in}$$

or

$$d_v \geq 0.9 d_e = 0.9 d_p = 0.9 (46.13) = 41.5 \text{ in}$$

Take  $d_v = 42.8$  inches

Then the critical section for shear  $x_{\text{vcrit}}$  is:

$$x_{\text{vcrit}} = (0.5 \cdot \text{sole plate length}) + d_v$$

$$= (0.5 \cdot 15.0) + 42.8$$

$$= 50.3 \text{ in} = 4.2 \text{ ft from centerline of bearing}$$

**Check Maximum Factored Shear Limit**

From Table 5.7.3.5 the Strength I design shear at 4.2 ft is

$$V_u = 285 \text{ kips}$$

The girder is supported by an integral type abutment at both ends. Therefore, the nominal shear capacity of the section is limited to:

**[5.7.3.3]**

$$V_n = 0.25 \cdot f'_c \cdot d_v \cdot b_v + V_p = 0.25 \cdot 9.5 \cdot 42.8 \cdot 6.5 + 0 = 661 \text{ kips}$$

The vertical prestress component,  $V_p$ , is set to zero in the above equation due to all straight strands.

The maximum design shear the section can have is:

$$\phi_v \cdot V_n = 0.90 \cdot 661 = 595 \text{ kips} > 285 \text{ kips} \quad \text{OK}$$

Note that if the girder was supported by a parapet type abutment or pier without a continuity diaphragm, which are not built integrally with its support, the shear stress would have been limited to  $0.18f'_c$  per AASHTO Article 5.7.3.2. Using an integral abutment, semi-integral abutment, or pier with a continuity diaphragm allows us to use the higher value from AASHTO Article 5.7.3.3.

**Determine Longitudinal Strain  $\epsilon_s$** 

Assume that minimum transverse reinforcement will be provided in the cross section. As previously noted,  $A_{ps}$  includes only the area of prestressing steel found on the flexural tension side of the member, as defined in LRFD Figure 5.7.3.4.2-1. At  $x_{vcrit}$ ,  $A_{ps}$  consists of only the bonded bottom flange strands.

Near the end of the beam and debonding,  $A_{ps}$  must also be reduced for development, so  $f_{ps}$  must be calculated again for the end section following the process shown previously:

$$\begin{aligned} f_{ps} &= f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right) = 300 \cdot \left(1 - 0.28 \cdot \frac{7.85}{46.13}\right) \\ &= 285.7 \text{ ksi} \end{aligned}$$

**[5.9.4.3]**

Strand development length  $\ell_d$  is:

$$\begin{aligned} \ell_d &= \kappa \cdot \left(f_{ps} - \frac{2}{3} f_{pe}\right) d_b \\ &= 1.6 \cdot \left[285.7 - \frac{2}{3} (164.7)\right] \cdot 0.6 = 168.9 \text{ in} \end{aligned}$$

Note that a  $\kappa$  value of 1.6 is used at the  $x_{vcrit}$  location because all acting strands at this location have no debonded length. When calculating

development where portions of the strand are debonded, a  $\kappa$  value of 2.0 is required per LRFD Article 5.9.4.3.3.

Transfer length  $\ell_{tr}$  is:

$$\ell_{tr} = 60 \cdot d_b = 60 (0.6) = 36.0 \text{ in}$$

At the critical section  $x_{vcrite} = (50.3 + 7.5) = 57.8$  inches, which alters the previous critical section measurement ( $x_{vcrit}$ ) from centerline of bearing to end of beam, the strand development fraction is:

$$\begin{aligned} F_{dev} &= \frac{f_{pe}}{f_{ps}} + \frac{x_{vcrite} - \ell_{tr}}{\ell_d - \ell_{tr}} \left( 1 - \frac{f_{pe}}{f_{ps}} \right) \\ &= \frac{164.7}{285.7} + \frac{57.8 - 36.0}{168.9 - 36.0} \left( 1 - \frac{164.7}{285.7} \right) = 0.65 \end{aligned}$$

Therefore,  $A_{ps} = A_{sps} \cdot F_{dev}$

$$= 6.51 \cdot 0.65 = 4.23 \text{ in}^2$$

#### [5.7.3.4.2]

Use LRFD equation 5.7.3.4.2-4 to compute the strain:

$$\begin{aligned} \epsilon_s &= \frac{\left( \frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po} \right)}{E_s \cdot A_s + E_p \cdot A_{ps}} \\ &= \frac{\left[ \frac{|1048 \cdot 12|}{42.8} + |285 - 0| - (4.23 \cdot 0.70 \cdot 300) \right]}{28,500 \cdot 4.23} = -0.00257 \end{aligned}$$

Because the value is negative, the strain will be recalculated using an additional concrete term:

From Figure 5.4.6.1 of this manual,  $A_{ct} = 435 \text{ in}^2$

$$\begin{aligned} \epsilon_s &= \frac{\left( \frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po} \right)}{E_c \cdot A_{ct} + E_s \cdot A_s + E_p \cdot A_{ps}} \\ &= \frac{\left[ \frac{|1048 \cdot 12|}{42.8} + |285 - 0| - (4.23 \cdot 0.70 \cdot 300) \right]}{4899 \cdot 435 + 28,500 \cdot 4.23} = -0.00014 \end{aligned}$$

Computed strain limits:

$$-0.0004 < -0.00014 < 0.006 \quad \text{OK}$$

Compute the tensile stress factor  $\beta$  using LRFD equation 5.7.3.4.2-1

$$\beta = \frac{4.8}{1 + 750 \cdot \varepsilon_s} = \frac{4.8}{1 + 750 \cdot (-0.00014)} = 5.36$$

Compute the angle  $\theta$  using equation 5.7.3.4.2-3

$$\theta = 29 + 3500\varepsilon_s = 29 + 3500 \cdot (-0.00014) = 28.51 \text{ degrees}$$

Compute the concrete contribution:

$$V_c = 0.0316 \cdot \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v = 0.0316 \cdot 5.36 \cdot \sqrt{9.5} \cdot 6.5 \cdot 42.8 = 145.2 \text{ kips}$$

The required steel contribution is

$$V_s = V_n - V_c - V_p = \frac{V_u}{\phi_v} - V_c - V_p = \frac{285}{0.90} - 145.2 = 171.5 \text{ kips}$$

Find the required spacing of double leg #4 stirrups:

$$s = \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_s} = \frac{2 \cdot 0.20 \cdot 60 \cdot 42.8 \cdot \cot(28.51)}{171.5} = 11.0 \text{ in}$$

Try double leg stirrups at a 11 inch spacing at the end of the beam.

$$A_v = \frac{0.4 \cdot 12}{11} = 0.44 \text{ in}^2 / \text{ft} \quad V_s = 171.9 \text{ kips}$$

### [5.7.2.5]

Check that the minimum transverse reinforcement requirement is satisfied:

$$\begin{aligned} \frac{A_{vmin}}{s} &= 0.0316 \cdot \lambda \cdot \sqrt{f'_c} \cdot \frac{b_v}{f_y} \\ &= 0.0316 \cdot 1.0 \cdot \sqrt{9.5} \cdot \frac{6.5}{60} \cdot 12 = 0.13 \frac{\text{in}^2}{\text{ft}} < 0.44 \frac{\text{in}^2}{\text{ft}} \quad \text{OK} \end{aligned}$$

### [5.7.2.6]

Check maximum permitted stirrup spacing at  $x_{vcrite}$ :

$$V_u = \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v} = \frac{285}{0.90 \cdot 6.5 \cdot 42.8} = 1.14 \text{ ksi}$$

$$V_{ulimit} = 0.125 \cdot f'_c = 0.125 \cdot 9.5 = 1.19 \text{ ksi} > 1.14 \text{ ksi}$$

Then the maximum spacing is the smaller of:

$$s_{\max} = 0.8 \cdot d_v = 0.8 \cdot 42.8 = 34.2 \text{ in}$$

$$\text{or } s_{\max} = 24 \text{ in} \quad \text{GOVERNS}$$

$$s_{\max} = 24 \text{ in} > 11 \text{ in} \quad \text{OK}$$

Therefore, use double leg #4 stirrups at 11 inch spacing. Other sections are investigated similarly.

#### [5.7.4]

### 2. Interface Shear Transfer

The standard beam details require that the outer 6 inches on each edge of the top flange will be smooth with a bond breaker applied, which leaves 22 inches of the top flange to be roughened for engagement of shear transfer.

Then  $b_{vi} = 22 \text{ in}$

Calculate  $d_{vi}$  as the distance between the centroid of the tension steel at the critical shear section to the mid-thickness of the slab.

$$d_{vi} = h_{\text{comp}} - y_{\text{sstr}} - \frac{t_s}{2} = 50 - 3.87 - \frac{8.5}{2} = 41.9 \text{ in}$$

The Strength I vertical shear at the critical shear section due to all loads is:

$$V_u = 285 \text{ kip}$$

$$v_{ui} = \frac{V_u}{b_{vi} \cdot d_{vi}} = \frac{285}{22 \cdot 41.9} = 0.31 \text{ ksi}$$

Interface shear force is:

$$V_{ui} = v_{ui} \cdot \frac{12 \text{ in}}{\text{ft}} \cdot b_{vi} = 0.31 \cdot 12 \cdot 22 = 81.8 \frac{\text{kip}}{\text{ft}}$$

Required nominal interface design shear is:

$$V_{n\text{ireq}} = \frac{V_{ui}}{\phi_v} = \frac{81.8}{0.90} = 90.9 \text{ kip/ft}$$

The interface area per 1 foot length of beam is:

$$A_{cv} = 22 \cdot 12 = 264.0 \text{ in}^2/\text{ft}$$

**[5.7.4.4]**

The standard beam details require the top flanges of the beam to be roughened. Then:

$$c = 0.28 \text{ ksi} \quad \mu = 1.0 \quad K_1 = 0.3 \quad K_2 = 1.8 \text{ ksi}$$

The upper limits on nominal interface shear are:

$$K_1 \cdot f'_c \cdot A_{cv} = 0.3 \cdot 4 \cdot 264.0 = 316.8 \text{ kip/ft} > 96.2 \text{ kip/ft} \quad \text{OK}$$

and

$$K_2 \cdot A_{cv} = 1.8 \cdot 264.0 = 475.2 \text{ kip/ft} > 96.2 \text{ kip/ft} \quad \text{OK}$$

The nominal interface shear resistance is:

$$V_{ni} = cA_{cv} + \mu(A_{vf} \cdot f_y + P_c)$$

$$P_c = 0.0 \text{ kip}$$

Substitute and solve for required interface shear steel:

$$A_{vfreq} = \frac{V_{nireq} - c \cdot A_{cv}}{\mu \cdot f_y} = \frac{90.9 - 0.28 \cdot 264.0}{1.0 \cdot 60} = 0.28 \text{ in}^2/\text{ft}$$

**[5.7.4.2]**

Check minimum interface shear requirements:

The minimum requirement may be waived for girder-slab interfaces with the surface roughened to an amplitude of 0.25 in if the factored interface shear stress is less than 0.210 ksi.

$$v_{ui} = 0.31 \text{ ksi} > 0.210 \text{ ksi}$$

Then the minimum requirement cannot be waived.

The minimum required interface shear reinforcement is the lesser of:

$$A_{vmin1} = \frac{0.05 \cdot b_v}{f_y} = \frac{0.05 \cdot 22}{60} = 0.018 \text{ in}^2/\text{in} = 0.22 \text{ in}^2/\text{ft}$$

or

$$\begin{aligned} A_{vmin2} &= \frac{1.33 \cdot V_{nireq} - c \cdot A_{cv}}{\mu \cdot f_y} = \frac{1.33 \cdot 90.9 - 0.28 \cdot 264}{1.0 \cdot 60} \\ &= 0.78 \text{ in}^2/\text{ft} \end{aligned}$$

$$\text{Then } A_{vmin} = 0.22 \text{ in}^2/\text{ft}$$

The double leg #4 stirrup at 11" spacing ( $A_v=0.44 \text{ in}^2/\text{ft}$ ) chosen earlier for vertical shear also meet the requirements for interface shear. Therefore, no additional reinforcement is required for interface shear.

Other sections are investigated similarly.

### [5.7.3.5]

### 3. Minimum Longitudinal Reinforcement Requirement

The longitudinal reinforcement must be checked to ensure it is adequate to carry the tension caused by shear. The amount of strand development must be considered near the end of the beam. There are 2 cases to be checked:

Case 1: From the inside edge of bearing at the end supports out to the critical section for shear, the following must be satisfied, with  $A_{ps} \cdot f_{ps}$  modified for development:

$$A_{ps} \cdot f_{ps} \geq \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot \theta$$

A crack starting at the inside edge of the bearing sole plate will cross the center of gravity of the strands at:

$$x_{\text{crack}} = L_{\text{soleplate}} + y_{\text{sstr}} \cdot \cot(\theta) = 15 + 3.87 \cdot \cot(28.51) = 22.1 \text{ in}$$

The transfer length for 0.6" strands is:  $\ell_{tr} = 36.0 \text{ in}$

From the end of the beam to full transfer length, the strand stress increases linearly from zero to  $f_{pe}$ . Interpolate to find the tensile capacity of the bonded strands at the intersection with the assumed crack:

$$T_{r1} = A_{ps} \cdot f_{pe} \cdot \frac{x_{\text{crack}}}{\ell_{tr}} = 30 \cdot 0.217 \cdot 164.7 \cdot \frac{22.1}{36} = 658 \text{ kips}$$

The tension force to carry is:

$$\begin{aligned} T_{u1} &= \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot(\theta) \\ &= \left( \frac{285}{0.90} - 0.5 \cdot 171.9 - 0 \right) \cdot \cot(28.51) \\ &= 424.8 \text{ kips} < 658 \text{ kips} \quad \text{OK} \end{aligned}$$

Case 2: At the critical section for shear, the following must be satisfied, with  $A_{ps} \cdot f_{ps}$  modified for development:

$$A_{ps} \cdot f_{ps} \geq \frac{M_u}{\phi_f d_v} + \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot(\theta)$$



Use values calculated earlier to determine the tensile capacity at the critical section for shear:

$$f_{ps} = 285.7 \text{ ksi}$$

$$A_{ps} = 6.51 \text{ in}^2$$

$$\text{Development fraction, } F_{dev} = 0.65$$

$$T_{r2} = A_{ps} \cdot f_{ps} \cdot F_{dev} = 6.51 \cdot 285.7 \cdot 0.65 = 1209 \text{ kips}$$

The factored moment  $M_u$  should be the moment concurrent with the factored shear  $V_u$  at  $x_{vcrit}$ . For simplicity, the maximum  $M_u$  at  $x_{vcrit}$  is used below.

Then the tension force to carry is:

$$\begin{aligned} T_{u2} &= \frac{M_u}{\phi_f d_v} + \left( \frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p \right) \cdot \cot(\theta) \\ &= \frac{1048 \cdot 12}{1.0 \cdot 42.8} + \left( \frac{285}{0.90} - 0.5 \cdot 171.9 - 0 \right) \cdot \cot(28.51) \end{aligned}$$

$$T_{u2} = 718.6 \text{ kips} < 1209 \text{ kips} \quad \text{OK}$$

**G. Design  
Pretensioned  
Anchorage Zone  
Reinforcement  
[5.9.4.4.1]**

**Splitting Reinforcement**

To prevent cracking in the beam end due to the transfer of the prestressing force from the strands to the concrete, splitting reinforcement needs to be provided in the anchorage zone.

Use a load factor of 1.0 and lateral force component of 4% of the fully bonded strands to determine the required amount of steel.

The total prestressing force at transfer of bonded strands:

$$P_i = A_{ps} \cdot (f_{pj} - \Delta f_{pES}) = 6.51 \cdot (216.0 - 25.0) = 1243 \text{ kips}$$

The factored design splitting force is:

$$P_{split} = 1.0 \cdot 0.04 \cdot P_i = 1.0 \cdot 0.04 \cdot 1243 = 49.7 \text{ kips}$$

The amount of resisting reinforcement is determined using a steel stress  $f_s$  of 20 ksi:

$$A_s = \frac{P_b}{f_s} = \frac{49.7}{20} = 2.49 \text{ in}^2$$

This steel should be located at the end of the beam within a distance of:

$$\frac{h}{4} = \frac{40}{4} = 10 \text{ in}$$

The number of #5 double legged stirrups necessary to provide this area is:

$$\frac{A_s}{2 \cdot A_b} = \frac{2.49}{2 \cdot 0.31} = 4.02$$

The first set of stirrups (G505E) is located 2 inches from the end of the beam. See Figure 5.7.3.5.

Provide an additional four sets of #5 stirrups (G508E) spaced at 2 1/2 inch centers.

$$x_{\text{splitting}} = 2 + 4 \cdot 2.5 = 12 \text{ in} > 10 \text{ in}$$

Although the splitting reinforcement does not fit within  $h/4$ , #5 bars are the largest allowed and 2.5 inches is the tightest spacing allowed. This is OK per MnDOT practice.

#### [5.9.4.4.2]

#### Confinement Reinforcement

Reinforcement is required at the ends of the beam to confine the prestressing steel in the bottom flange. G303E bars (see Figure 5.7.2.5) will be placed at a maximum spacing of 6 inches out to  $1.5d$  from the ends of the beam. For simplicity in detailing and ease of tying the reinforcement, space the vertical shear reinforcement with the confinement reinforcement in this area.

$$1.5d = 1.5 \cdot 40 = 60.0 \text{ in}$$

#### H. Determine Camber and Deflection

##### [2.5.2.6.2]

##### [3.6.1.3.2]

##### [5.6.3.5.2]

#### Camber Due to Prestressing and Dead Load Deflection

Using the memo to designers #2023-01, the camber due to prestress can be found. First, the strands must be separated into groups based on their debonded length. Then the eccentricity and prestress force just after transfer can be used to determine each group's contribution to release camber. The span length at release is the end-to-end length of the 119.25 feet since the beam is in the casting bed. Using the following equations, calculate the upward deflection values due to prestressing strand in Table 5.7.3.8.

Force in the strand:

$$P_t = A_{ps} \cdot (f_{pj} - \Delta f_{pES})$$

Camber due to prestressing strands:

$$\Delta_{ps_{total}} = \sum_{i=1}^n \Delta_{ps_i}$$

$$\Delta_{ps_i} = \frac{P_{t_i} \cdot e_{s_i} \cdot L^2}{8 \cdot E_{ci} \cdot I} \quad \text{(Straight Bonded Strands)}$$

$$\Delta_{ps_i} = \frac{P_{t_i} \cdot e_{s_i} \cdot [L^2 - (L_t + 2 \cdot L_{x_i})^2]}{8 \cdot E_{ci} \cdot I} \quad \text{(Debonded Strand)}$$

Where:

$\Delta_{ps_{total}}$  = upward camber of beam immediately after release, due to prestress alone (in)

$\Delta_{ps_i}$  = upward camber contribution immediately after release, due to individual strand group (in)

$P_{t_i}$  = prestress force immediately after release of individual strand group (kips)

$e_{s_i}$  = eccentricity of prestress force with respect to the beam centroid at midspan of individual strand group (in)

$L$  = end to end length of beam (in)

$L_t$  = transfer length of strand (in)

$L_{x_i}$  = length of debonding from end of beam of individual strand group (in)

$E_{ci}$  = modulus of elasticity of concrete at prestress transfer (ksi)

$I$  = beam moment of inertia (in<sup>4</sup>)

**Table 5.7.3.8**

**Camber at Release Due to Prestressing Strands**

Group	$L_x$ (in)	$e_s$ (in)	$A_{ps}$ (in <sup>2</sup> )	$P_t$ (k)	$\Delta_{ps}$ (in)
1	0	14.20	6.51	1243	6.62 ①
2	168	12.74	1.30	248	1.11 ②
3	216	14.07	1.30	248	1.17 ②
4	264	16.07	1.30	248	1.26 ②
Sum					10.16

① Calculated using "Straight Bonded Strands" equation.

② Calculated using "Debonded Strands" equation.

Downward deflection due to selfweight

$$\Delta_{sw} = \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I} = \frac{5 \cdot \frac{0.758}{12} (119.25 \cdot 12)^4}{384 \cdot 4578 \cdot 149,002} = 5.06 \text{ in}$$

Camber at release  $\Delta_{rel} = \Delta_{ps} + \Delta_{sw} = 10.16 - 5.06 = 5.10$  in

To estimate camber at the time of erection the deflection components are multiplied by standard MnDOT multipliers. They are:

Release to Erection Multipliers:

Prestress = 1.4

Selfweight = 1.4

Camber and selfweight deflection values at erection are:

Prestress:	$1.4 \cdot 10.16 = 14.22$ in
Selfweight:	$1.4 \cdot (-5.06) = -7.08$ in
Diaphragm DL:	-0.02 in
Deck and stool DL:	-5.12 in
Barrier:	-0.37 in

Note that the deflection values for diaphragms, deck, stool, and barrier are based on a span length of 118.0 feet.

The values to be placed in the camber diagram on the beam plan sheet are arrived at by combining the values above.

$$\text{"Erection Camber"} = 14.22 - 7.08 - 0.02 = 7.12 \text{ in} \quad \text{say } 7 \frac{1}{8} \text{ in}$$

$$\text{"Est. Dead Load Deflection"} = 5.12 + 0.37 = 5.49 \text{ in} \quad \text{say } 5 \frac{1}{2} \text{ in}$$

$$\text{"Est. Residual Camber"} = 7 \frac{1}{8} - 5 \frac{1}{2} = 1 \frac{5}{8} \text{ in}$$

### Live Load Deflection

The deflection of the bridge is checked when subjected to live load and compared against the limiting values of  $L/800$  for vehicle only bridges and  $L/1000$  for bridges with bicycle or pedestrian traffic.

Deflection due to lane load is:

$$\Delta_{\text{lane}} = \left( \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I} \right) = \left[ \frac{5 \cdot \frac{0.64}{12} \cdot (118 \cdot 12)^4}{384 \cdot 4899 \cdot 396,823} \right] = 1.44 \text{ in}$$

Deflection due to a truck with dynamic load allowance is found using hand computations or computer tools to be:

$$\Delta_{\text{truck}} = 2.81 \text{ in}$$

Two deflections are computed and compared to the limiting values, that of the truck alone and that of the lane load plus 25% of the truck. Both deflections need to be adjusted with the live load distribution factor for deflection.

$$\Delta_1 = DF_{\Delta} \cdot \Delta_{\text{truck}} = 0.425 \cdot 2.81 = 1.19 \text{ in}$$

$$\Delta_2 = DF_{\Delta} \cdot (\Delta_{\text{lane}} + 0.25 \cdot \Delta_{\text{truck}}) = 0.425 \cdot (1.44 + 0.25 \cdot 2.81) = 0.91 \text{ in}$$

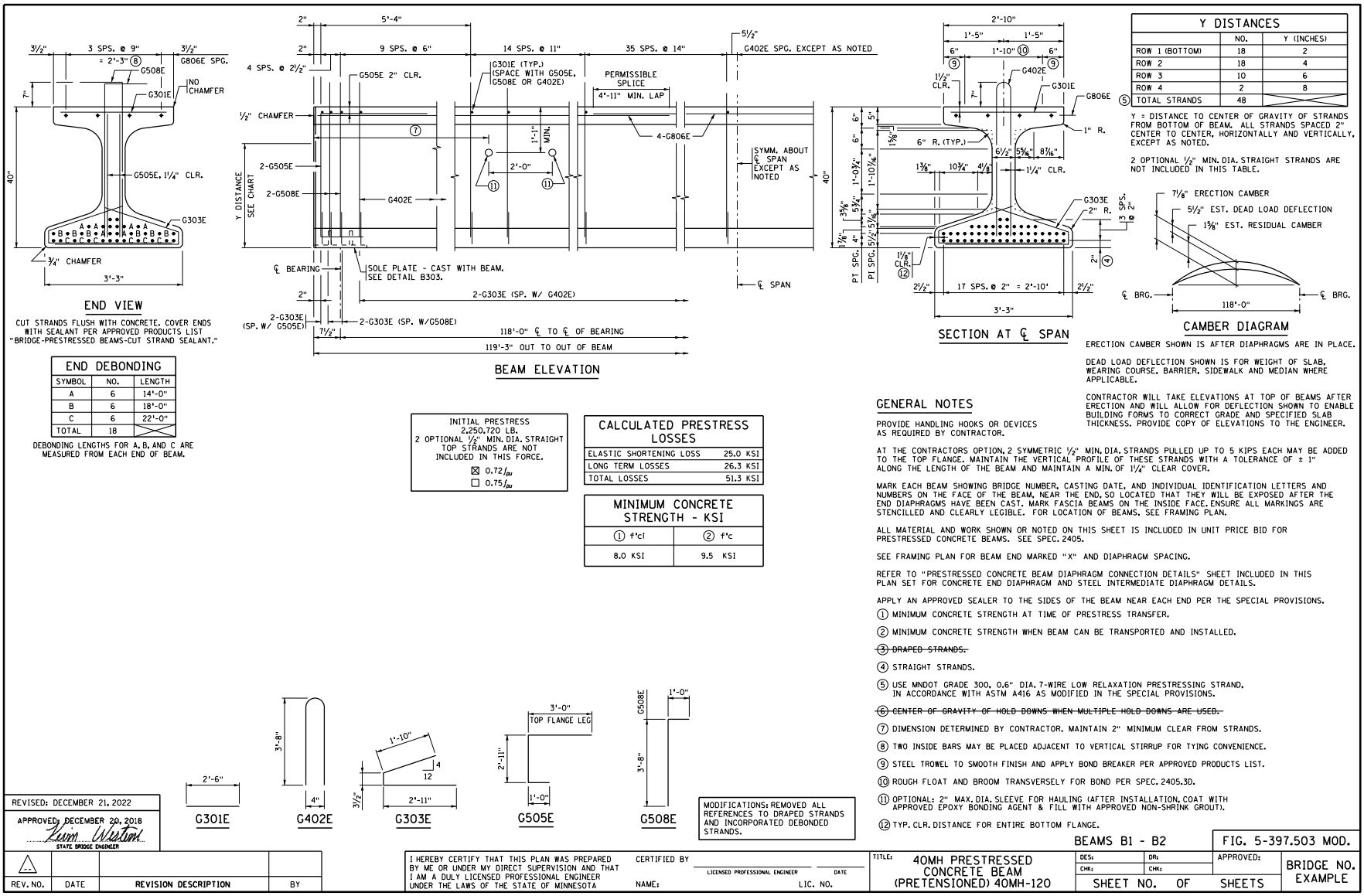
There is no bicycle or pedestrian traffic on the bridge, so the deflection limit is:

$$\frac{L}{800} = \frac{118 \cdot 12}{800} = 1.77 \text{ in} > \text{ than } \Delta_1 \text{ or } \Delta_2 \quad \text{OK}$$

***I. Beam Sheet for  
Bridge Plan***

Figure 5.7.3.6 shows the detailed beam sheet for the debonded strand configuration that will be included in the bridge plan.

Figure 5.7.3.6



**5.7.4 Three-Span  
Haunched Post-  
Tensioned Concrete  
Slab Design  
Example**

[Future manual content]



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**APPENDIX 5-A**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $\ell_d$ .

**DECKS:**

**Top Transverse Deck Bars**

See LRFD Bridge Design Manual Table 9.2.1.1 or Table 9.2.1.2 for bar size and spacing. A Class A splice is provided where all top transverse bar splices occur between beams, with 50% of the bars spliced at a given location. A Class B splice is provided where 100% of the bars are spliced at a given location between beams or where 50% of the bars are spliced at a given location over beams. Avoid splicing 100% of bars over beams.

<b>Top Transverse Deck Bar Lap Splice Lengths</b>				
Concrete Cover to Bar Being Considered	Bar Spacing	Bar Size	All Splices are Between Beams and 50% are at Same Location ( <i>preferred</i> ) (Class A)	100% of Splices at Same Location Between Beams or 50% of Splices at Same Location Over Beams (Class B)
3"	> 5"	#4	1'-6"	1'-11"
		#5	1'-10"	2'-5"
		#6	2'-2"	2'-10"
	5"	#4	1'-6"	1'-11"
		#5	1'-10"	2'-5"
		#6	2'-9"	3'-7"

**Top Longitudinal Deck Bars**

See LRFD Bridge Design Manual Table 9.2.1.1 & Figure 9.2.1.8 or Table 9.2.1.2 & Figure 9.2.1.9 for bar size and spacing. Detail reinforcement such that no more than 50% of top longitudinal bars are spliced at any cross-section through the deck (Class A splice).

<b>Top Longitudinal Deck Bar Lap Splice Lengths</b>		
Concrete Cover to Bar Being Considered	Bar Size	Lap Splice Length (Class A)
$\geq 3 \frac{1}{2}$ "	#4	1'-6"
	#5	1'-10"
	#6	2'-2"

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $l_d$ .

**DECKS: (cont'd)**

**Bottom Transverse Deck Bars**

See LRFD Bridge Design Manual Table 9.2.1.1 or Table 9.2.1.2 for bar size and spacing. A Class A splice is provided where all bottom transverse bars are spliced over beams, with 50% of the bars spliced at a given location. A Class B splice is provided where 100% of the bars are spliced at a given location over beams or where 50% of the bars are spliced at a given location between beams. Avoid splicing 100% of bars between beams.

<b>Bottom Transverse Deck Bar Lap Splice Lengths</b>				
Concrete Cover to Bar Being Considered	Bar Spacing	Bar Size	All Splices are Over Beams and 50% are at Same Location ( <i>preferred</i> ) (Class A)	100% of Splices at Same Location Over Beams or 50% of Splices at Same Location Between Beams (Class B)
1"	≥ 4"	#4	1'-10"	2'-5"
		#5	2'-9"	3'-6"
		#6	3'-9"	4'-10"

**Bottom Longitudinal Deck Bars**

See LRFD Bridge Design Manual Table 9.2.1.1 or Table 9.2.1.2 & Figure 9.2.1.9 for bar size and spacing. A Class B splice is provided. Where possible, detail such that no more than 50% of the bottom longitudinal deck bars are spliced at a given cross-section through the deck.

<b>Bottom Longitudinal Deck Bar Lap Splice Lengths</b>			
Concrete Cover to Bar Being Considered	Bar Spacing	Bar Size	50% of Splices at Same Location ( <i>Preferred</i> ) or 100% of Splices at Same Location (Class B)
≥ 1 1/2"	≥ 4"	#4	1'-11"
		#5	3'-0"
		#6	3'-7"

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $l_d$ .

**ABUTMENTS:**

**Abutment and Wingwall Vertical Bars**

Back face vertical bars are all spliced at the same location, so a Class B splice is used. See LRFD 5.10.8.4.3a. Front face bars are conservatively assumed to act as tension reinforcement, so compressive development lengths are not used in splice length computations. Although all front face bars are spliced at the same location, excess reinforcement is provided, so a Class A splice is used.

<b>Abutment and Wingwall Vertical Bar Lap Splice Lengths</b>					
Concrete Cover to Bar Being Considered	Bar Size	Back Face Bar Spacing (Class B)			Front Face Bar Spacing (Class A)
		4"	5"	≥6"	≥6"
≥ 2"	#4	--	--	--	1'-6"
	#5	3'-0"	2'-5"	2'-5"	1'-10"
	#6	3'-7"	3'-7"	3'-7"	2'-9"
	#7	4'-6"	4'-2"	4'-2"	3'-2"
	#8	5'-11"	4'-9"	4'-9"	3'-8"
	#9	7'-6"	6'-0"	5'-10"	--
	#10	9'-6"	7'-7"	7'-2"	--
	#11	11'-8"	9'-4"	8'-8"	--
	#14	--	13'-5"	11'-10"	--

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $\ell_d$ .

**ABUTMENTS: (cont'd)**

**Abutment and Wingwall Horizontal Bars**

All horizontal bars are assumed to have more than 12" of concrete cast below. For integral abutment stem and diaphragm and for semi-integral abutment diaphragm, horizontal bars resist passive pressure loads, so a Class B splice is used. For parapet abutment stem and backwall and semi-integral abutment stem, horizontal bars are assumed to provide excess reinforcement, so a Class A splice is used. For long wingwalls on separate footings, horizontal bars become primary reinforcement, so a Class B splice is used.

<b>Abutment and Wingwall Horizontal Bar Lap Splice Lengths</b>					
Concrete Cover to Bar Being Considered	Bar Size	Integral Abutment Stem & Diaphragm and Semi-Integral Abutment Diaphragm Horizontal Bar Spacing (Class B)	Parapet Abutment Stem & Backwall and Semi-Integral Abutment Stem Horizontal Bar Spacing (Class A)	Wingwall Horizontal Bar Spacing (Class B)	
		≥ 6	≥ 6"	4"	≥ 5"
≥ 2"	#4	2'-6"	1'-11"	2'-6"	2'-6"
	#5	3'-1"	2'-5"	3'-4"	3'-1"
	#6	4'-0"	3'-1"	4'-0"	4'-0"
	#7	4'-8"	3'-7"	5'-1"	4'-8"
	#8	5'-4"	4'-1"	6'-8"	5'-4"

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $\ell_d$ .

**PIERS:**

**Pier Cap Top Longitudinal Bars**

All horizontal bars are assumed to have more than 12" of concrete cast below. For splices between columns (or between piles for a pile bent pier) where no more than 50% of the bars are spliced at the same location, a Class A splice is used. For all other cases, use a Class B splice.

<b>Pier Cap Top Longitudinal Bar Lap Splice Lengths</b>							
Concrete Cover to Bar Being Considered	Bar Size	All Splices Located Between Columns and $\leq 50\%$ of Bars Are Spliced at Same Location (Class A)					
		Bar Spacing					
		4"	5"	5 1/2"	6"	$\geq 6 \frac{1}{2}$ "	
$\geq 2 \frac{3}{8}$ "	#5	2'-7"	2'-5"	2'-5"	2'-5"	2'-5"	
	#6	3'-1"	3'-1"	2'-10"	2'-10"	2'-10"	
	#7	3'-11"	3'-7"	3'-7"	3'-7"	3'-7"	
	#8	5'-2"	4'-1"	4'-1"	4'-1"	4'-1"	
	#9	6'-6"	5'-3"	4'-9"	4'-8"	4'-8"	
	#10	8'-3"	6'-7"	6'-0"	5'-6"	5'-6"	
	#11	10'-2"	8'-2"	7'-5"	6'-10"	6'-8"	
	#14	--	11'-9"	10'-8"	9'-9"	9'-1"	
	Bar Size	All Splices Located Between Columns and $> 50\%$ of Bars Are Spliced at Same Location (Class B)					
		Bar Spacing					
		4"	5"	5 1/2"	6"	$\geq 6 \frac{1}{2}$ "	
		#5	3'-4"	3'-1"	3'-1"	3'-1"	3'-1"
		#6	4'-0"	4'-0"	3'-8"	3'-8"	3'-8"
		#7	5'-1"	4'-8"	4'-8"	4'-8"	4'-8"
		#8	6'-8"	5'-4"	5'-4"	5'-4"	5'-4"
		#9	8'-6"	6'-9"	6'-2"	6'-0"	6'-0"
		#10	10'-9"	8'-7"	7'-10"	7'-2"	7'-2"
		#11	13'-3"	10'-7"	9'-8"	8'-10"	8'-7"
#14	--	15'-3"	13'-10"	12'-9"	11'-10"		

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $\ell_d$ .

**PIERS: (cont'd)**

**Pier Cap Bottom Longitudinal Bars**

For splices over columns (or over piles for a pile bent pier) where no more than 50% of the bars are spliced at the same location, a Class A splice is used. For all other cases, use a Class B splice.

<b>Pier Cap Bottom Longitudinal Bar Lap Splice Lengths</b>							
Concrete Cover to Bar Being Considered	Bar Size	All Splices Located Over Columns and $\leq 50\%$ of Bars Are Spliced at Same Location (Class A)					
		Bar Spacing					
		4"	5"	5 1/2"	6"	$\geq 6\ 1/2$ "	
$\geq 2\ 3/8$ "	#5	2'-3"	1'-10"	1'-10"	1'-10"	1'-10"	
	#6	2'-9"	2'-9"	2'-2"	2'-2"	2'-2"	
	#7	3'-6"	3'-2"	3'-2"	3'-2"	3'-2"	
	#8	4'-6"	3'-8"	3'-8"	3'-8"	3'-8"	
	#9	5'-9"	4'-7"	4'-2"	4'-1"	4'-1"	
	#10	7'-4"	5'-10"	5'-4"	4'-11"	4'-10"	
	#11	9'-0"	7'-2"	6'-7"	6'-0"	5'-10"	
	#14	--	10'-4"	9'-5"	8'-8"	8'-1"	
	Bar Size	All Splices Located Over Columns and $> 50\%$ of Bars Are Spliced at Same Location (Class B)					
		Bar Spacing					
		4"	5"	5 1/2"	6"	$\geq 6\ 1/2$ "	
		#5	3'-0"	2'-5"	2'-5"	2'-5"	2'-5"
		#6	3'-7"	3'-7"	2'-10"	2'-10"	2'-10"
		#7	4'-6"	4'-2"	4'-2"	4'-2"	4'-2"
		#8	5'-11"	4'-9"	4'-9"	4'-9"	4'-9"
		#9	7'-6"	6'-0"	5'-5"	5'-4"	5'-4"
		#10	9'-6"	7'-7"	6'-11"	6'-4"	6'-4"
		#11	11'-8"	9'-4"	8'-6"	7'-10"	7'-7"
#14	--	13'-5"	12'-3"	11'-3"	10'-5"		

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $\ell_d$ .

**PIERS: (cont'd)**

**Other Pier Cap Longitudinal Bars Located on Side Faces of Pier Cap**

Longitudinal bars located on the side faces of pier caps (typically skin or shrinkage and temperature reinforcement) are assumed to have more than 12" of concrete cast below. For these bars, a Class B splice is used.

Lap Splice Lengths for Longitudinal Bars Located on Side Faces of Pier Cap		
Concrete Cover to Bar Being Considered	Bar Size	Bar Spacing $\geq 4"$ (Class B)
$\geq 2 \frac{3}{8}"$	#4	2'-6"
	#5	3'-4"
	#6	4'-0"
	#7	5'-1"

**Pier Column Vertical Bars**

For pier columns, all splices occur at the same location, so a Class B splice is used.

Pier Column Vertical Bar Lap Splice Lengths						
Concrete Cover to Bar Being Considered	Bar Size	Bar Spacing (Class B)				
		4"	5"	5 1/2"	6"	$\geq 6 \frac{1}{2}"$
$\geq 2 \frac{3}{8}"$	#6	3'-7"	3'-7"	2'-10"	2'-10"	2'-10"
	#7	4'-6"	4'-2"	4'-2"	4'-2"	4'-2"
	#8	5'-11"	4'-9"	4'-9"	4'-9"	4'-9"
	#9	7'-6"	6'-0"	5'-5"	5'-4"	5'-4"
	#10	9'-6"	7'-7"	6'-11"	6'-4"	6'-4"
	#11	11'-8"	9'-4"	8'-6"	7'-10"	7'-7"
	#14	--	13'-5"	12'-3"	11'-3"	10'-5"

**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $l_d$ .

**SLAB SPANS:**

**Top Bars**

This table applies to both top longitudinal and transverse bars. All bars are assumed to have more than 12" of concrete cast below. A Class B splice is used.

<b>Top Longitudinal and Transverse Bar Lap Splice Lengths</b>						
Concrete Cover to Bar Being Considered	Bar Size	Bar Spacing (Class B)				
		4"	5"	6"	7"	≥ 8"
≥ 3"	#4	2'-6"	2'-6"	2'-6"	2'-6"	2'-6"
	#5	3'-4"	3'-1"	3'-1"	3'-1"	3'-1"
	#6	4'-0"	4'-0"	3'-8"	3'-8"	3'-8"
	#7	5'-1"	4'-8"	4'-8"	4'-4"	4'-4"
	#8	6'-8"	5'-4"	5'-4"	4'-11"	4'-11"
	#9	8'-6"	6'-9"	6'-0"	6'-0"	6'-0"
	#10	10'-9"	8'-7"	7'-2"	6'-9"	6'-9"
	#11	13'-3"	10'-7"	8'-10"	7'-7"	7'-6"
	#14	--	15'-3"	12'-9"	10'-11"	9'-11"



**APPENDIX 5-A (CONTINUED)**

**MnDOT BRIDGE OFFICE REBAR LAP SPLICE GUIDE**

- Notes: > Splice lengths are based on BDM Tables 5.2.2.1 and 5.2.2.2.
- > Assumes use of epoxy coated bars. Excess reinforcement factor  $\lambda_{er} = 1.0$ .
- > Class A splice length is equivalent to the rebar development length,  $\ell_d$ .

**SLAB SPANS: (cont'd)**

**Bottom Bars**

The table applies to both bottom longitudinal and transverse bars. A Class B splice is used.

<b>Bottom Longitudinal and Transverse Bar Lap Splice Lengths</b>			
Concrete Cover to Bar Being Considered	Bar Size	Bar Spacing (Class B)	
		4"	≥ 5"
≥ 1 1/2"	#4	1'-11"	1'-11"
	#5	3'-0"	3'-0"
	#6	3'-7"	3'-7"
	#7	4'-8"	4'-8"
	#8	5'-11"	5'-11"
	#9	7'-6"	7'-3"
	#10	9'-6"	8'-11"
	#11	11'-8"	10'-7"
	#14	--	14'-4"

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